7.1 NTH Roots : Rational Exponents

$\boxed{\sqrt[n]{a}}$ set up

$n = n^{th}$ root

$a =$ any #

ex $\sqrt[2]{16} = 16^{1/2}$ $\sqrt[3]{8} = a^{1/3}$

$= 14$ $= 2$

$\sqrt[4]{1} = 1^{\frac{1}{4}}$

P1 Finding n^{th} Roots

ie1 $n = 3, a = -125$

$\sqrt[3]{-125} = \boxed{-5}$

ie2 $n = 4, a = 16$

$\sqrt[4]{16}$

$= \pm 2$

Rational Exponents

- $a^{m/n} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$

- $a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$

pwr

radi

P2 Evaluating Expressions w/ Rational Exponents

ie 1

P2 Evaluating Expressions w/ Rational Exponents

ie1 $9^{3/2} = \left(\sqrt[2]{9}\right)^3 = 3^3 = 27$

ie2 $32^{-2/5} = \dfrac{\left(\sqrt[5]{32}\right)^2}{1} = \dfrac{1}{\left(\sqrt[5]{32}\right)^2} = \dfrac{1}{2^2} = \dfrac{1}{4}$

ie3 $\left(\sqrt[4]{5}\right)^3 = 5^{\frac{3}{4}} = 3.34$

~~ie4~~ $\cancel{2x^4}$

P4 Solving Equations Using nth Roots

ie1 $\frac{2x^4}{2} = \frac{162}{2}$

$\sqrt[4]{x^4} = \sqrt[4]{81}$

$x^1 = \pm 3$

$x^{\frac{4}{4}} =$

ie2 $\sqrt[3]{(x-2)^3} = \sqrt[3]{10}$

$x - 2 = \sqrt[3]{10}$

$\quad +2 \quad +2$

$x = 2 + \sqrt[3]{10}$

$x = 2 + 10^{1/3}$

$x = 4.15$

ie3 $\frac{6\cancel{x}^4}{6} = \frac{3750}{6}$

$\sqrt[4]{x^4} = \sqrt[4]{625}$ $\qquad 625^{1/4}$

$x = \pm 5$

ie4 $\sqrt[3]{(x+1)^3} = \sqrt[3]{18}$ $\qquad = 18^{1/3}$

$x+1 = \sqrt[3]{18}$

$\quad -1 \qquad\quad -1$

$x = \sqrt[3]{18} - 1$

$x \approx 1.621$

7.2 Properties of Rational Exponents

Property	Example
1.) $a^m \cdot a^n = a^{m+n}$	$2^2 \cdot 2^3 = 2^{2+3} = 2^5$
2.) $(a^m)^n = a^{m \cdot n}$	$(2^2)^3 = 2^{2 \cdot 3} = 2^6$
3.) $(ab)^m = a^m b^m$	$(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 36$
4.) $a^{-m} = \frac{1}{a^m}$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
5.) $\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^2}{3^1} = 3^{2-1} = 3$
6.) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

ie1 $5^{1/2} \cdot 5^{1/4} = 5^{\left(\frac{1}{2}+\frac{1}{4}\right)} = 5^{\frac{2}{4}+\frac{1}{4}} = 5^{\frac{3}{4}}$

ie2 $\left(8^{1/2} \cdot 5^{1/3}\right)^2 = 8^{\frac{1}{2}\cdot\frac{2}{1}} \cdot 5^{\frac{1}{3}\cdot\frac{2}{1}}$

$= 8^{\frac{2}{2}} \cdot 5^{\frac{2}{3}} = 8 \cdot 5^{\frac{2}{3}}$

ie3 $\left(2^4 \cdot 3^4\right)^{-\frac{1}{4}} = 2^{\left(\frac{4}{1}\cdot\frac{-1}{4}\right)} \cdot 3^{\left(\frac{4}{1}\cdot\frac{-1}{4}\right)} = 2^{\frac{-4}{4}} \cdot 3^{\frac{-4}{4}}$

$= 2^{-1} \cdot 3^{-1} = \frac{1}{2^1} \cdot \frac{1}{3^1} = \frac{1}{6}$

ie4 $\frac{7}{7^{\frac{1}{3}}} = \frac{7^{\frac{3}{3}}}{7^{1/3}} = 7^{\frac{3}{3}-\frac{1}{3}} = 7^{\frac{2}{3}}$

ie5 $\left(\frac{12^{1/3}}{4^{1/3}}\right)^2 = \frac{12^{\frac{1}{3}\cdot\frac{2}{1}}}{4^{\frac{1}{3}\cdot\frac{2}{1}}} = \frac{12^{\frac{2}{3}}}{4^{\frac{2}{3}}} = \left(\frac{12}{4}\right)^{\frac{2}{3}} = 3^{\frac{2}{3}}$

P2 Using Properties of Radicals

ie1 $\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{4 \cdot 16} = \sqrt[3]{64} = 4$

ie2 $\dfrac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\dfrac{162}{2}} = \sqrt[4]{81} = 3$

P3 Writing Radicals in Simplest Form

ie1 $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2}$

$\boxed{3\sqrt[3]{2}}$

ie2 $\sqrt[5]{\dfrac{3}{4}} = \dfrac{\sqrt[5]{3}}{\sqrt[5]{4}} \cdot \dfrac{\sqrt[5]{8}}{\sqrt[5]{8}} = \dfrac{\sqrt[5]{24}}{\sqrt[5]{32}} = \boxed{\dfrac{\sqrt[5]{24}}{2}}$

P4 Adding & Subtract. Roots & Radicals

ie1 $7\left(6^{\frac{1}{5}}\right) + 2\left(6^{\frac{1}{5}}\right) = (7+2)\left(6^{\frac{1}{5}}\right)$

$= 9\left(6^{1/5}\right)$

ie2 $\sqrt[3]{16} - \sqrt[3]{2} =$

$\sqrt{16}$

$\sqrt{2}\cdot\sqrt{8}$

$\sqrt{2}\cdot\sqrt{4}$

$\sqrt{2}\cdot\sqrt{2}$

$2\sqrt[3]{2} - 1\sqrt[3]{2}$

$= 1\sqrt[3]{2}$

P5 Simplifying Expressions Involving Variables

ie1 $\sqrt[3]{125y^6} =$

y^6

$y^2 \cdot y^3$

$\sqrt{125}$

$\sqrt{5} \; \sqrt{25}$

$\sqrt{5} \; \sqrt{5}$

$5y\sqrt{y^2} \Leftarrow \sqrt[3]{5^3 y^3 y^2}$

ie2 $(9u^2v^{10})^{\frac{1}{2}}$

$= 9^{1/2} u^{\frac{2}{1}\cdot\frac{1}{2}} \cdot v^{\frac{10}{1}\cdot\frac{1}{2}}$

or $(\sqrt[2]{9})^1$

$= 3 \cdot u^1 \cdot v^5$

ie3 $\sqrt[4]{\frac{x^4}{y^8}} = \sqrt[4]{\frac{x^4}{y^4 \cdot y^4}} = \frac{x}{y^2}$

ie4 $\frac{6xy^{1/2}}{2x^{1/3}z^{-5}} = \frac{6xy^{\frac{1}{2}}}{2x^{1/3}z^{-5}}$

$\frac{6x^{3/3}y^{\frac{1}{2}}z^{+5}}{2x^{1/3}\cancel{z}^{1}} = 3x^{\frac{2}{3}}y^{\frac{1}{2}}z^{5}$

P6 Writing Variable Expressions in Simplest Form:

ie 1 $\sqrt[5]{5a^5b^9c^{13}} = \sqrt[5]{5^1 \cdot a^5 \cdot b^5 \cdot b^4 \cdot c^5 \cdot c^5 \cdot c^3}$

$= abc^2\sqrt[5]{5b^4c^3}$

ie 2 $\sqrt[3]{\frac{x}{y^7}} = \frac{\cancel{\sqrt[3]{x^1}}}{\cancel{\sqrt[3]{y^3 \cdot y^3 \cdot y^1}}} \rightarrow = \frac{\sqrt[3]{x^1}}{\sqrt[3]{y^3 \cdot y^3 \cdot y^1}}$

$\frac{\sqrt[3]{x^1 \cdot y^2}}{y^2 \sqrt[3]{y^1}} \Rightarrow \frac{\sqrt[3]{x^1}}{y^2\sqrt[3]{y^1}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}$

$\frac{\sqrt[3]{xy^2}}{y^2\sqrt{y^1 \cdot y^2}} = \frac{\sqrt[3]{xy^2}}{y^2\sqrt[3]{y^3}}$

$= \sqrt[3]{xy^2}$

e 2.

P7 Adding / Subtracting Expressions Involving Variables

ie1 $5\sqrt{y} + 6\sqrt{y} = (5+6)(\sqrt{y}) = 11\sqrt{y}$

ie2 $2xy^{1/3} - 7xy^{\frac{1}{3}} = (2-7)(xy^{1/3})$

$-5xy^{1/3}$

ie3 $3\sqrt[3]{5x^5} - x\sqrt[3]{40x^2}$

$\sqrt{40}$

$\sqrt{2} \cdot \sqrt{20}$

$\sqrt{2} \cdot \sqrt{10}$

$\sqrt{2} \cdot \sqrt{5}$

$\cancel{BAX}: 3\sqrt[3]{5x^5}$

$= 3\sqrt[3]{5 \cdot x^3 \cdot x^2} - x\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5 \cdot x^2}$

$3x\sqrt[3]{5x^2} - 2x\sqrt[3]{5x^2} = 3x - 2x\sqrt[3]{5x}$

$1x\sqrt[3]{5x^2}$

7.3 Alg. 2 Power Functions ; Function Operations

Addition $f(x) + g(x) = h(x)$

Subtract $f(x) - g(x) = h(x)$

Mult. $f(x) \cdot g(x) = h(x)$

Division $\dfrac{f(x)}{g(x)} = h(x)$

(P1) Adding / Subtract Functions: $f(x) = 2x^{1/2}$, $g(x) = -6x^{1/2}$

(ie1) $f(x) + g(x)$

$2\underline{\underline{x}}^{1/2} + -6\underline{\underline{x}}^{1/2} = -4x^{\frac{1}{2}}$

must be same

ie 2 $f(x) - g(x)$

$2x^{1/2} - (-6x^{1/2})$

$\underbrace{2x^{1/2} + 6x^{\frac{1}{2}}}_{\text{must be same}}$

$8x^{1/2}$

P2 Mult / Divide Functions

$f(x) = 3x$

$g(x) = x^{1/4}$

ie 1 $f(x) \cdot g(x) = 3x(x^{1/4})$

$3x^{4/4}(x^{\frac{1}{4}})$

$3x^{5/4}$

* Add powers with 2 bases

Remember

$x^1 = x^{\frac{1}{1}}$ n $x^{\frac{2}{2}}$

or $x^{3/3}$ or $x^{4/4}$

ie 2 $\dfrac{f(x)}{g(x)} = \dfrac{3x}{x^{1/4}} = \dfrac{3x^{\frac{4}{4}}}{x^{1/4}}$

$= 3x^{\frac{4}{4}-\frac{1}{4}} = 3x^{3/4}$

Composition: when you complete 2 operations or more @ 1 interval of time.

P3 Finding composite of functions

- $\underset{1st}{f} \circ \underset{2nd}{g}(x) = f(g(x))$
- $\overset{1st}{g} \circ \overset{2nd}{f}(x) = g(f(x))$

* 2nd listed function goes into (substituted) into first function

Let $f(x) = 3x^{-1}$; $g(x) = 2x-1$

$\boxed{\text{ie1}}$ $f(g(x)) = f \circ \underline{g}(x)$

$f(x) = 3x^{-1}$

$f(x) = 3(2x-1)^{-1}$

$= \dfrac{3}{(2x-1)^{1}}$

ie2 $g(f(x)) =$

$g(x) = 2x-1$

$g(x) = 2(3x^{-1})-1$

$= 6x^{-1}-1$

$= \dfrac{6}{x} - 1$

ie3 $f(f(x))$

$f(x) = 3x^{-1}$

$= 3(3x^{-1})^{-1}$

$= 3^{1}(3^{-1}x^{1})$

$= 3^{1-1}x^{1}$

$= x$

7.4 Finding Inverse (ALG2)

Reflection

Inverse Relation: means the domain (x) and range (y) exchange locations.

(ex)

x	y
-2	1
-3	2
2	3
4	4

inverse $\xrightarrow{rm}$

x	y
1	-2
2	-3
3	2
4	4

In an equation switch x ! y

ex1 $y = 2x - 4$ $\xrightarrow{\text{inverce then solve for } y}$ $x = 2y - 4$

$$x = 2y - 4$$
$$-2y \quad -2y$$
$$-2y + x = -4$$
$$-x \quad -x$$
$$\frac{-2y}{-2} = \frac{-x}{-2} \frac{-4}{-2}$$
$$y = \tfrac{1}{2}x + 2$$

P2

Inverse Functions (Proving them as inverses must equal same thing X)

$f(g(x)) = x$ $\quad$ $g(f(x)) = x$

$f \circ g = x$ $\quad$ $g \circ f = x$

Verifying Inverse Functions

ie1 $f(x) = 2x - 4$ $\quad$ $g(x) = \frac{1}{2}x + 2$

$f \circ g(x)$

$f(x) = 2\left(\frac{1}{2}x + 2\right) - 4$

$f(x) = x + 4 - 4$

$f(x) = x$

$g \circ f$

$g(x) = \frac{1}{2}(2x - 4) + 2$

$g(x) = x - 2 + 2$

$g(x) = x$

ie 2 $f(x) = -3x+6$

$f \circ g(x)$

$f^{-1}(x) = -\frac{1}{3}x + 2$ ← inverse

$f^{-1}(x)$ or $g \circ f(x)$

$f(x) = -3\left(-\frac{1}{3}x+2\right)+6$

$f(x) = +x-6+6$

$f(x) = x$

$g(x)$ or $f^{-1}(x) = -\frac{1}{3}(-3x+6)+2$

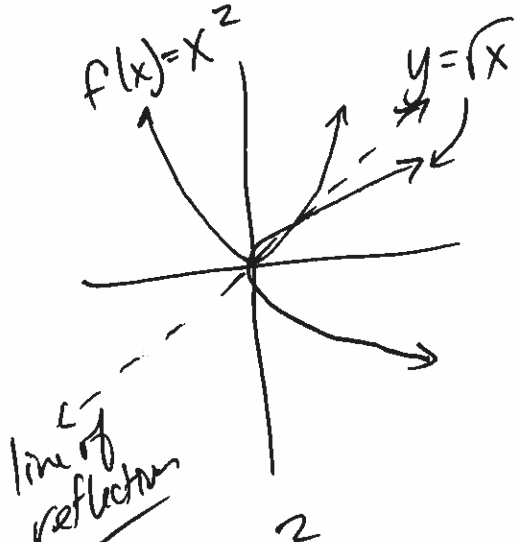
$x-2+2$

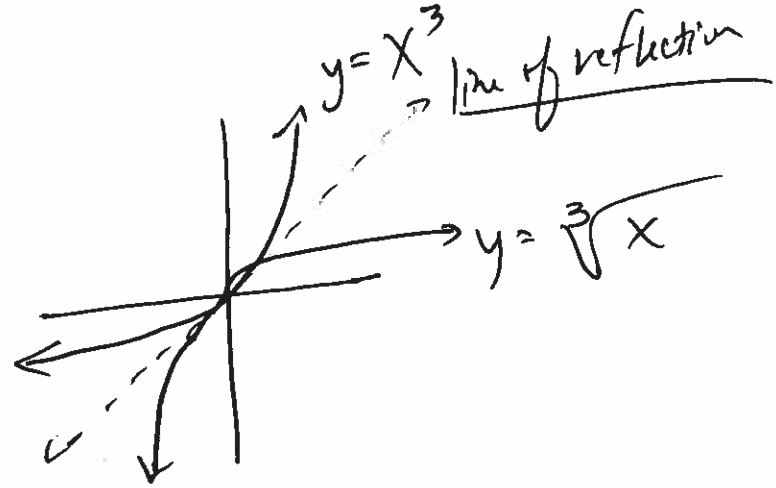
$f^{-1}(x) = x$

or $g(x$

(P3) Finding Inverses of Non-linear Functions

(ie1) $f(x) = x^2$ (ie2) $g(x) = x^3$





$y = x^2$

$\Rightarrow \sqrt{x} = \sqrt{y^2}$

inverse

$\boxed{y = \sqrt{x}}$ inverse

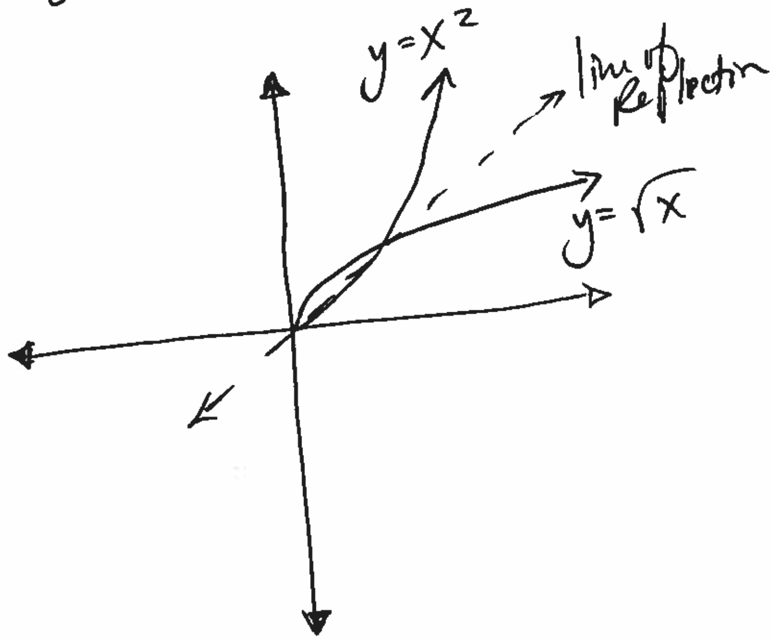
$y = x^3$

$\Rightarrow \sqrt[3]{x} = \sqrt[3]{y^3}$

inverse

$\boxed{y = \sqrt[3]{x}}$ inverse

ie2 $f(x) = x^2$; $\boxed{x \geq 0}$ Restriction



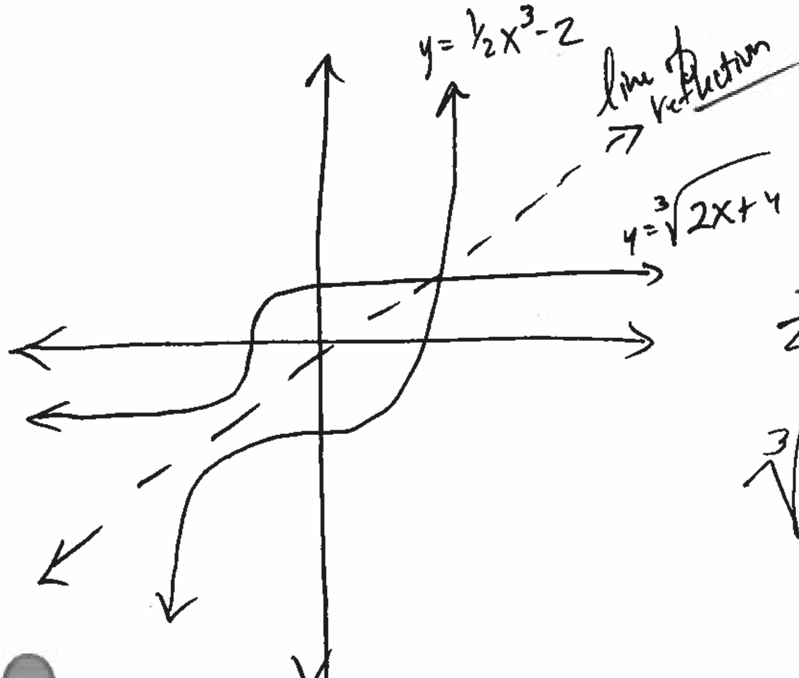
inverse

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$y = \sqrt{x}$

ie3 $f(x) = \frac{1}{2}x^3 - 2$



Inverse

$y = \frac{1}{2}x^3 - 2$

$x = \frac{1}{2}y^3 - 2$
$+2 \qquad\qquad +2$

$2(x+2) = \left(\frac{1}{2}\right)y^3 \, 2$

$\sqrt[3]{2x+4} = \sqrt[3]{y^3}$

$y = \sqrt[3]{2x+4}$

7.4 HWK pgs. 426-427 ALG2 # 1-31 all

1.) horizontal line does not cross the graph

2) Reflection of one anom

3.) switch X & y in original equation

4)

X	y
-1	1
-2	2
-3	3
-4	4
-5	5

5.)

X	y
2	-4
1	-2
0	0
1	2
2	4

6.) $y = \frac{x}{5}$

7.) $y = \frac{x+1}{2}$

8.) $y = \frac{-3}{2}x + 9$

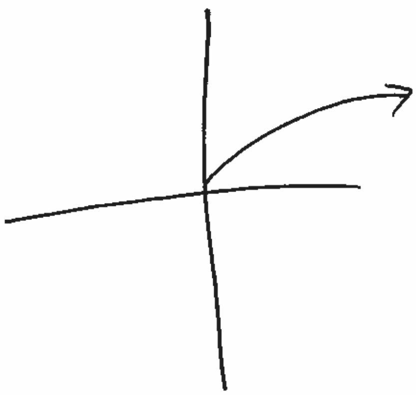
9.) Both equal X

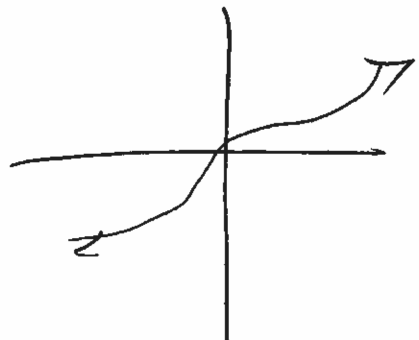
10.) Both equal X

11.) $\frac{\sqrt[4]{27x}}{3}$

7.5 Graphing Square Root & Cubic Root Function

Radical Functions $y=\sqrt{x}$ & $y=\sqrt[3]{x}$





Formula $y=a\sqrt{x-h}+k$ or $y=a\sqrt[3]{x-h}+k$

h - shifts graph horizontally

k - shift graph up & dow

- a - amplitude (opening)

P1 Comparing 2 Graphs

~~parent $y=\sqrt{x}$~~ changes

ie1 $y=\sqrt{x+1}-3$; $y=\sqrt{x}$ parent

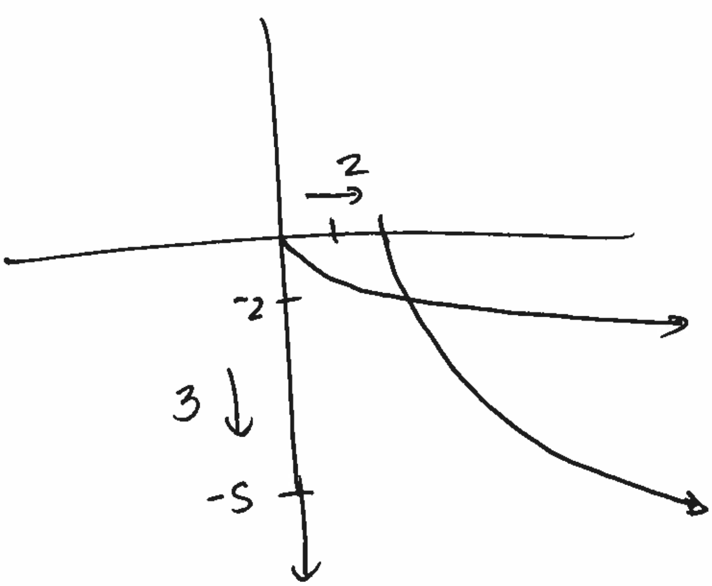
$h=-1$ (1 unit left)

$K=-3$ (3 units down)

$h=0$

$K=0$

ie2 $y=-3\sqrt{x-2}+1$; $y=\sqrt{x}$ parent



$h=2$ (2 right)

$K=1$ (up 1 unit)

$a=-3$ (negative end behavior arrow)

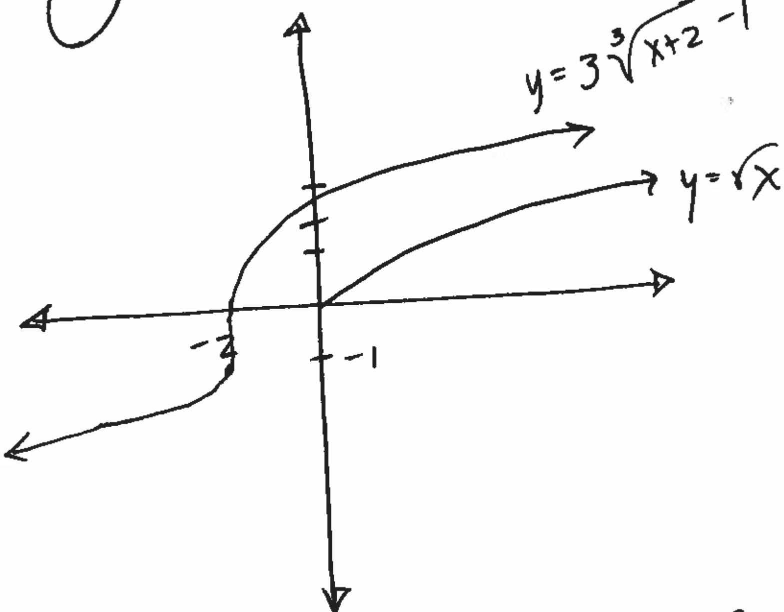
ie3 $y = 3\sqrt[3]{x+2} - 1$

parent $y = \sqrt{x}$

$h = -2$

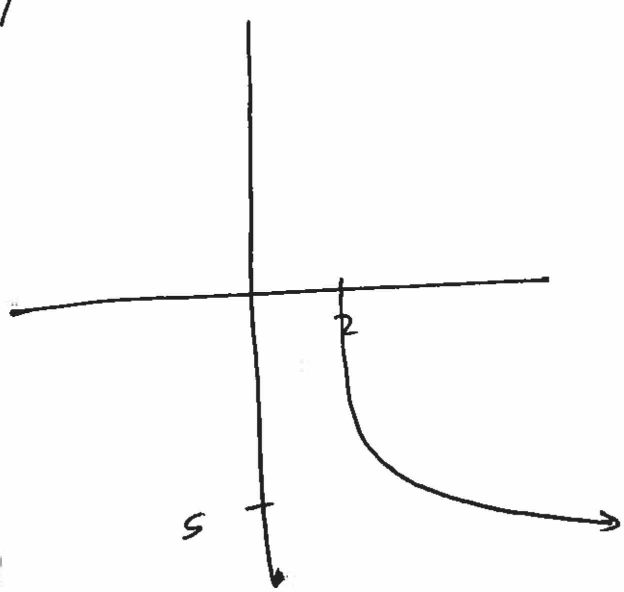
$K = -1$

$a = 3$ (positive)



ie4 Finding Domain : Range

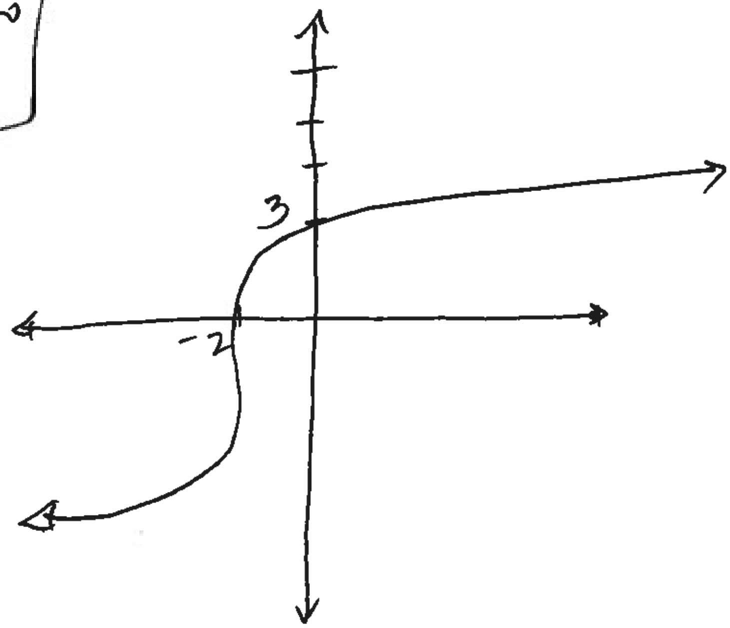
go back to ie 2



D: $\{2, \infty)$

R: $\{0; 5\}$

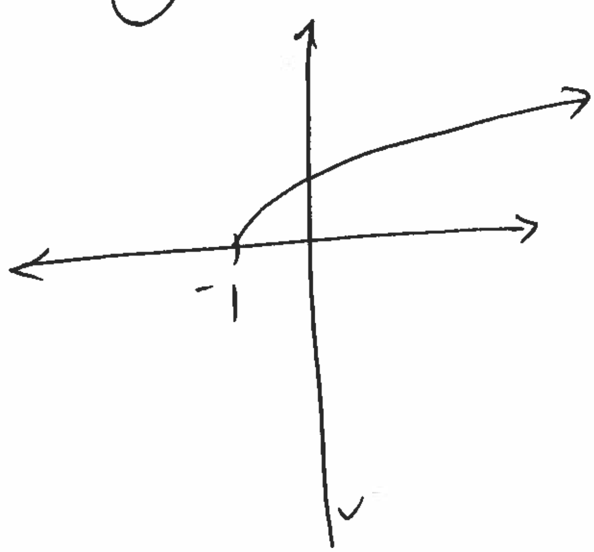
back to ie3



D: $\{-\infty, \infty)$

R: $\{-\infty, \infty)$

ie3 $y = \sqrt{x+1}$



D: $\{-\cancel{1}, \infty)$

R: $\{0, \infty)$

7.6 Solving Radical Equations:

P1 Solving with Radical Exponents

ie1 $\sqrt[3]{x} - 4 = 0$

$\quad\; +4 \;\; +4$

$\left(\sqrt[3]{x}\right)^3 = (4)^3$

$x = 4^3$

$x = 64$

ie2 $\frac{2x^{\frac{3}{2}}}{2} = \frac{250}{2}$

$\left(x^{3/2}\right)^{2/3} = (125)^{2/3}$

$x = 25$

ie3 $\sqrt{4x-7} + 2 = 5$

$\quad\quad\quad\; -2 \;\; -2$

$\left(\sqrt{4x-7}\right)^2 = (3)^2$

$4x - 7 = 9$

$\quad\; +7 \;\; +7$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4$

P2 Solving Equation W/2 Radicals

ie1 $\sqrt{3x+2} - 2\sqrt{x} = 0$

$\left(\cancel{\sqrt{3x+2}} + 2\sqrt{x}\right)^{2} = (0)^{2}$

$+2\sqrt{x}$

$\left(\sqrt{3x+2}\right)^2 = \left(2\sqrt{x}\right)^2$

$3x+2 = 4x$

$-3x \qquad -3x$

$2 = x$

ie 2

$$\sqrt{4x+28} - 3\sqrt{2x} = 0$$

$$+3\sqrt{2x} \quad +3\sqrt{2x}$$

$$\left(\sqrt{4x+28}\right)^2 = \left(3\sqrt{2x}\right)^2$$

$$4x+28 = 3^2 \cdot 2x$$

$$4x+28 = 9 \cdot 2x$$

$$4x+28 = 18x$$

$$-4x \qquad -4x$$

$$\frac{28}{14} = \frac{14x}{14}$$

$$x = 2$$

P3 Equation with Extraneous Solution

extraneous Solution: Not a valid Solution

ic1 $(X-4)^2 = (\sqrt{2X})^2$

$(X-4)(X-4) = 2X$

$X^2 - 8X + 16 = 2X$
$\quad -2X \qquad\quad -2X$

$X^2 - 10X + 16 = 0$

$(x-8)(x-2) = 0$

$X = 8$

$X = 2$ Extraneous Solution

Check:

$\cancel{8} - 4 = \sqrt{2(8)}$

$4 = \sqrt{16}$

$4 = 4$

$2 - 4 = \sqrt{2(2)}$

$-2 = \sqrt{4}$

$-2 \neq 2$

ONLY ANSWER is $X = 8$

(4)

ie2 $(X-3)^2 = (\sqrt{4X})^2$

$(X-3)(X-3) = 4X$

$X^2 - 6X + 9 = 4X$

$ -4X \qquad\quad -4X$

$X^2 - 10X + 9 = 0$

$(X-9)(X-1) = 0$

$X = 9$

$X = 1$

$(9-3) = \sqrt{4(9)}$

$6 = \sqrt{36}$

$6 = 6$ OK

$1 - 3 = \sqrt{4(1)}$

$-2 = 2$ no

Extraneous Solution

$\boxed{X = 9}$

7.6 HWK ALG 2 pg. 441 #17-46

17.) $\sqrt{x}-3=6;\ x=81$

yes

18.) NO

20.) yes

19.) yes

22.) yes

21.) NO

24.) $\frac{8}{125}$

23.) 4

26.) -3,200,000

25.) 27

27.) $\frac{4x^{3/4}}{4}=\frac{108}{4}$

$\left(x^{3/4}\right)^{4/3}=(27)^{4/3}$

$x=81$

28.) $\left(\left((x-4)^{3/2}\right)^{2/3}=(-6)^{2/3}\right)$

No solution $\left(\sqrt{-6}\right)^3$

$\cancel{x=4}$