2.1 Linear and Quadratic Functioning & Modeling

Polynomial functions: a function with a degree or power
real #; positive

$f(x)$

a) $4x^3-5x-\frac{1}{2}$

= degree of 3

Leading coefficient = 4

b) $g(x)=6x^{-4}+7$ not a polynomial

~~degrees~~

degree = -4

c) $h(x)=\sqrt{9x^4+16x^2}$ not polynomial

Cannot be simplified out of $\sqrt{\ }$ sign

d) $K(x)=15x-2x^4$

degree = 4

Leading coefficient = -2

Polynomial Functions of No and Low Degree

Zero Function $f(x) = 0$ degree undefined

Constant Function $f(x) = a \ (a \neq 0)$ degree $= 0$

Linear Function $f(x) = ax + b \ (a \neq 0)$ degree $= 1$

Quad. Function $f(x) \ ax^2 + bx + c$ degree 2

Linear Functions & Their Graphs

* Vertical Lines are not function b/c they fail vertical line test

* Slant Lines: neither horizontal or vertical

Finding an Equation of a Linear Function

ie1 Write linear function f such that

$$f(-1)=2 \; ; \; f(3)=-2$$

$\boxed{\text{step 1}}$ Find slope (or rate of change)

$$\underset{x_1}{(-1},\underset{y_1}{2)} \; ; \; \underset{x_2}{(3},\underset{y_2}{-2)}$$

FORMULA

$$\boxed{m=\frac{\Delta y}{\Delta x}} = \frac{-2-2}{3+(-1)} = \frac{-4}{4} = -1$$

$\boxed{\text{Step 2}}$ use point slope formula

$$\boxed{y-y_1=m(x-x_1)}$$

$$\underset{x_1}{(-1},\underset{y_1}{2)} \quad m=-1$$

$$y-2=-1(x-(-1))$$

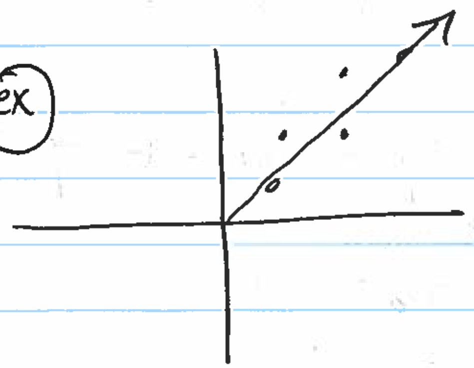
$$y-2=-1(x+1) \rightarrow \begin{array}{l} y-2=-x-1 \\ \;\;\;+2 \qquad\;\; +2 \end{array}$$

$$\boxed{y=-x+1}$$

Linear Correlation & Modeling

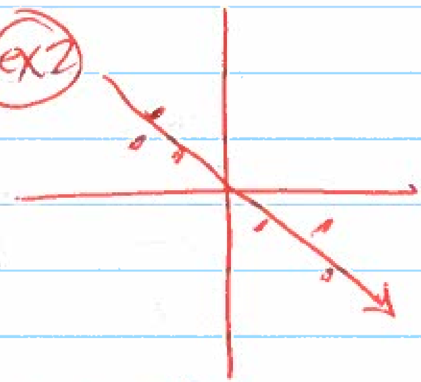
linear correlation: points clustered along a line on a scatter plot

ex



positive correlation

ex2

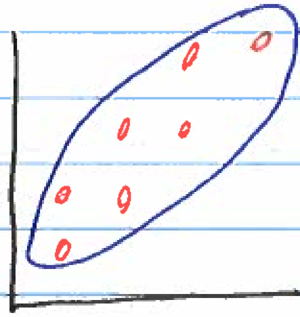


negative correlation

Linear correlation coefficient, r

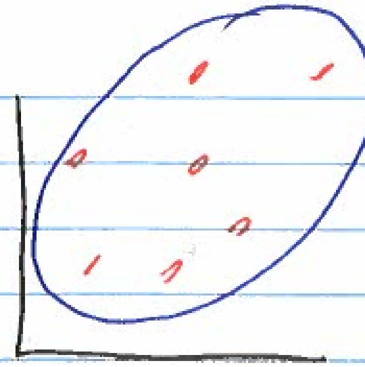
1.) $-1 \leq r \leq 1$ (when r falls the most)
2.) $r > 0$ positive corr.
3.) $r < 0$ neg. corr.
4.) $|r| \approx 1$ strong linear corr.
5.) $r \approx 0$ weak or no corr.

ex1



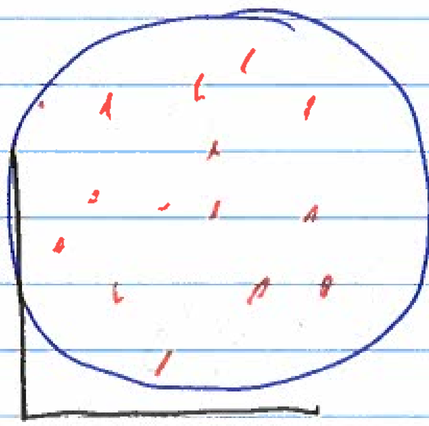
Strong positive correlation

ex2



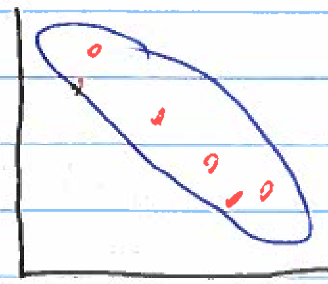
Weak positive linear corr.

ex3



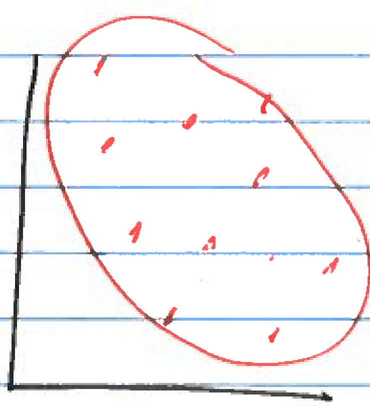
Little or no correlation

ex4



Strong neg. corr.

ex5



Weak neg. corr.

Quadratic Functions & Their Graphs

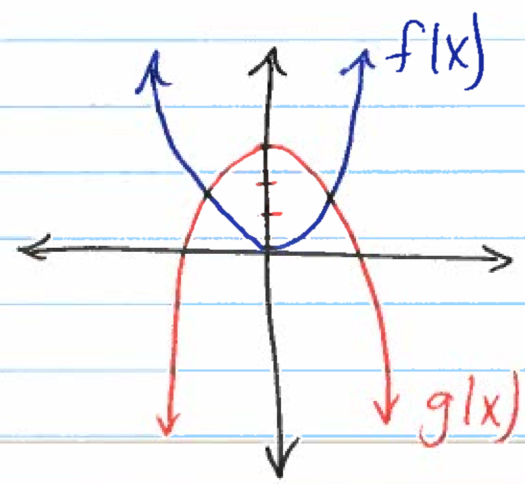
quad. function: degree of 2

$f(x) = x^2$ ← parent function

Transforming the Squaring Function

ie 1 $f(x) = x^2 \xrightarrow{\text{into}} g(x) = -(\tfrac{1}{2})x^2 + 3$

1) Vertically shrinking by $\tfrac{1}{2}$
2) Reflect the result across x-axis
3) translate 3 units up

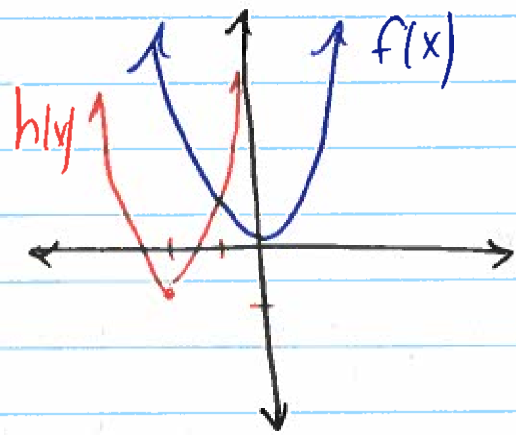


ie 2 $f(x) = x^2 \Longrightarrow h(x) = 3(x+2)^2 - 1$

① Vertically stretching graph by a factor of 3

② then move graph 2 units left

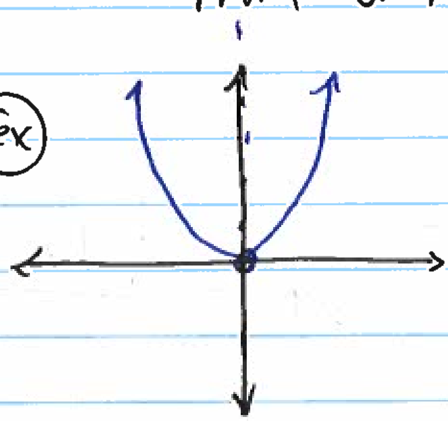
③ the move graph 1 unit down



Axis of Symmetry (A.S.): line of symmetry in a parabola

Vertex: point ~~on~~ parabola that is the min. or max on a parabola.

ex



A.S. = y-axis

Vertex = (0,0)

Standard quadratic form: (expanded form)

$$ax^2+bx+c$$

ex $f(x) = a(x-h)^2 + K$ ⟸ Vertex form

Stretch or Shrink

left/right

up/down

$f(x) = \cancel{ax}a$

$a(x-h)(x-h) + K$

$a(x^2-2hx+h^2)+K$

$ax^2 - 2ahx + ah^2 + K$

$\cancel{=ax^2 - 2ah}$

~~tex~~

FORMULA

Quad function

$$\boxed{x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}}$$

Finding the Vertex & Axis of a Quad Function

ie 1 $f(x) = 6x - 3x^2 - 5$

Reorder $f(x) = \underline{-3}x^2 + \underline{6}x \underline{-5}$

$a = -3$
$b = 6$
$c = -5$

$$\boxed{x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}}$$ FORMULA

$$x = \frac{-(6)}{2(-3)} \pm \frac{\sqrt{(6)^2 - 4(-3)(-5)}}{2(-3)}$$

$$x = \frac{-6}{-6} \pm \frac{\sqrt{36 - 30}}{-6}$$

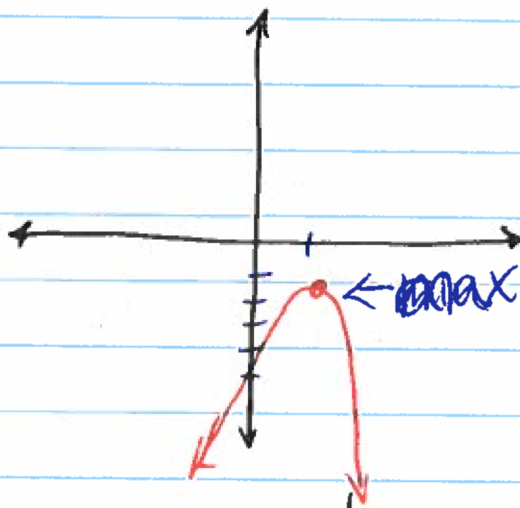
$$x = 1 \pm \frac{\sqrt{6}}{-6}$$

$$x = 1 + \frac{\sqrt{6}}{-6} \quad \& \quad x = 1 - \frac{\sqrt{6}}{-6}$$

or graph first

Graph first:

$$f(x) = -3x^2 + 6x - 5$$



To find vertex easy way!

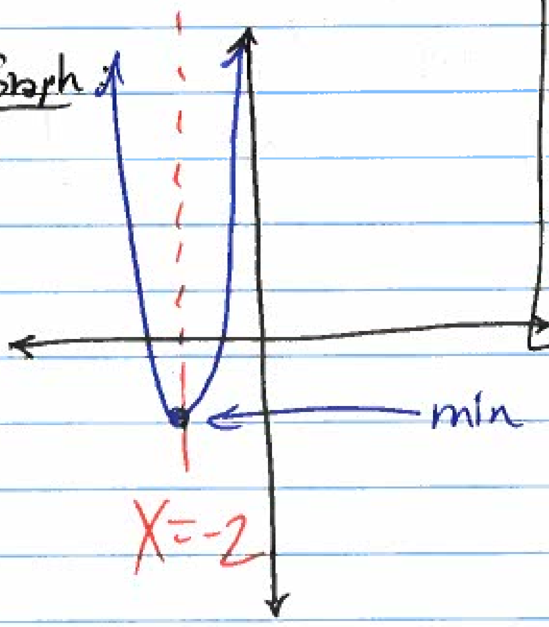
1. 2nd calc
2. min enter
3. line X on each side of vertex (max) entr entr entr

w/ calculator!

$\boxed{\text{Vertex: } (1, -2) \quad \text{A.S: } X = 1}$

1c2 $y = 3x^2+12x+11$

Graph:



Vertex: $(-2,-1)$

A.S: $x=-2$

#39 in HWK
with the
class

2.2. Power Functions w/ Modeling

Power Function: $f(x) = K \cdot x^a$

a - power
K - constant
$f(x)$ - function

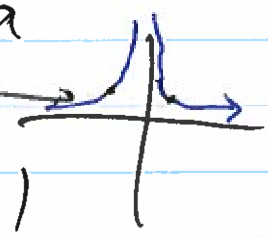
Common Power Functions

Name	Formula	power	Constant
Circumference	$C = 2\pi r$	1	2π
Area of Circle	$A = \pi r^2$	2	π
Force of Gravity	$F = K/d^2$	-2	K
Boyle's Law	$V = K/P$	-1	K

Direct Variation: powers that are positive

Inverse Variation: powers that are negative

~~Writing~~ Analyzing a Power Function Formula



ex 1 $f(x)=\sqrt[3]{x}$

graph

$f(x) = x^{\frac{1}{3}} = 1 \cdot x^{\frac{1}{3}}$

power = $1/3$
constant = 1
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuous
Increase for all x
Symmetric to origin
(odd function)
Not Bounded above or below
Local min or max
No Asymptotes
End Behaviors

$\lim_{x \to -\infty} \sqrt[3]{x} = -\infty$ $\quad \lim_{x \to +\infty} \sqrt[3]{x} = \infty$

ex 2 $g(x) = \frac{1}{x^2}$

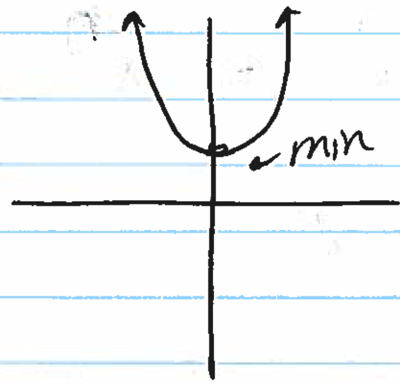
$g(x) = x^{-1/2} = 1 \cdot x^{-\frac{1}{2}}$

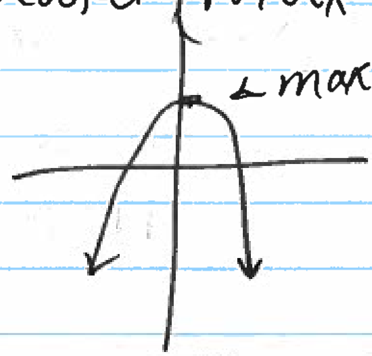
power = $-1/2$
Constant = 1
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(0, \infty)$
Continuous on domain
discontinuous $x=0$
Increase: $(-\infty, 0)$
Decrease $(0, \infty)$
Symmetric to y-axis
(even function)
Bounded below (not above)
No extrema
HA $y=0$
VA $x=0$
End behaviors

$\lim_{x \to -\infty} 1/x^2 = 0$

$\lim_{x \to \infty} (1/x^2) = 0$

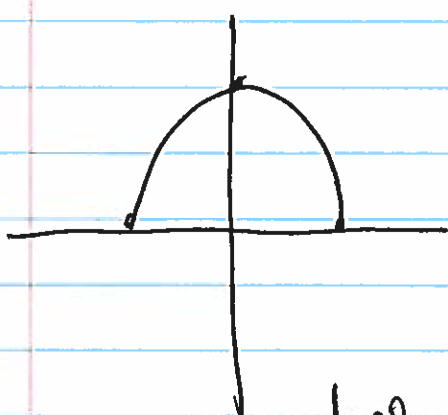
Bounded - more explanation
Where the boundary
is located (max or min)

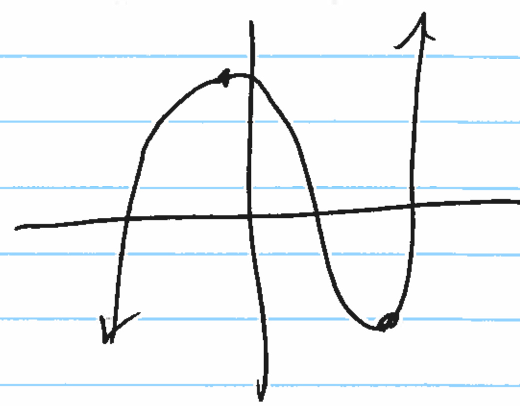




Bounded below
means the extrema
is below the graph
arrows

Bounded Above
means extrema
is above the
graph arrows





Bounded (no arrows)
(limits)

Not bounded due to a min & max present

Monomial Functions & Their Graphs

$f(x) = K$ or $f(x) = K \cdot x^n$

K - constant
n - positive integer

ex $f(x) = x$ $\quad g(x) = x^3$

$h(x) = x^5$

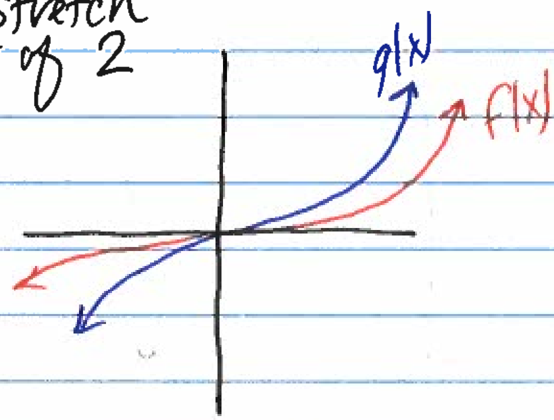
ex 2 $f(x) = x^2$ $\quad g(x) = x^4$

$h(x) = x^6$

Graphing Monomial Functions

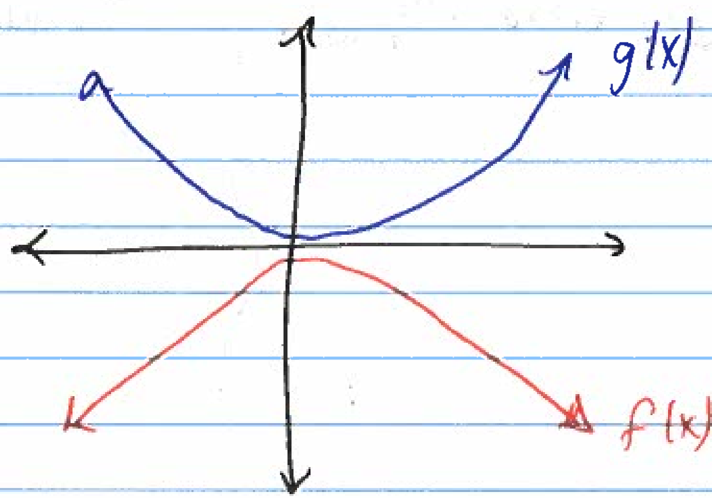
ie1 $f(x) = 2x^3$ parent: $g(x) = x^3$

Vertical stretch is factor of 2



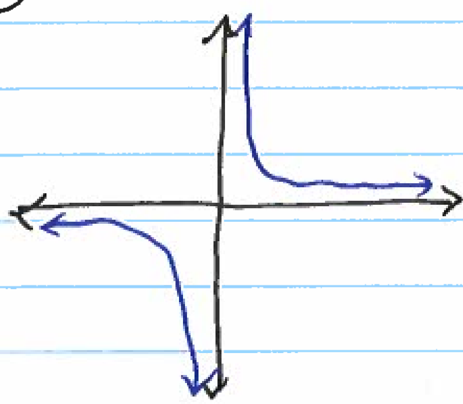
ie2 $f(x) = -\frac{2}{3}x^4$ parent: $g(x) = x^4$

Vertical shrink of 2/3



Graph Power Functions:

ie 1 $f(x) = 2x^{-3}$



$K = 2$

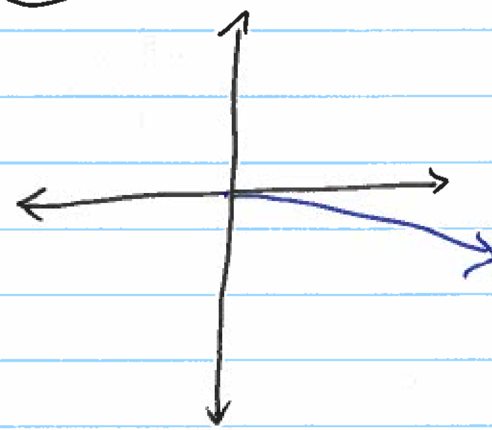
$a = -3$

Function: Odd

Symmetric to origin

Decreasing 1^{st} Quad.

ie 2 $f(x) = -0.4x^{1.5}$



$K = -0.4$

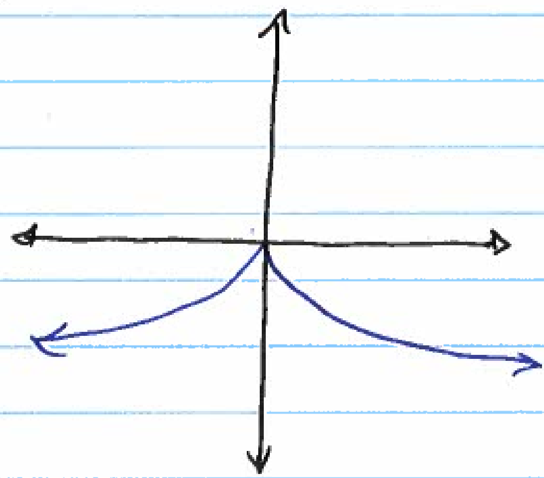
$a = 1.5$

Function: Constant

No symmetry

Decreasing 4^{TH} Quad

ie3 $f(x) = -x^{0.4}$



$K = -1$

$a = 0.4$

Function: Even

Symmetric: y-axis

Decreasing: 4^{TH} Quad.

do #27 with Student

2.3 Polynomial Functions of Higher Degree Modeling

<u>Cubic functions</u>: polynomial degree of 3.

<u>quartic functions</u>: polynomial degree of 4.

<u>Standard form</u>: polynomial written in descending degree

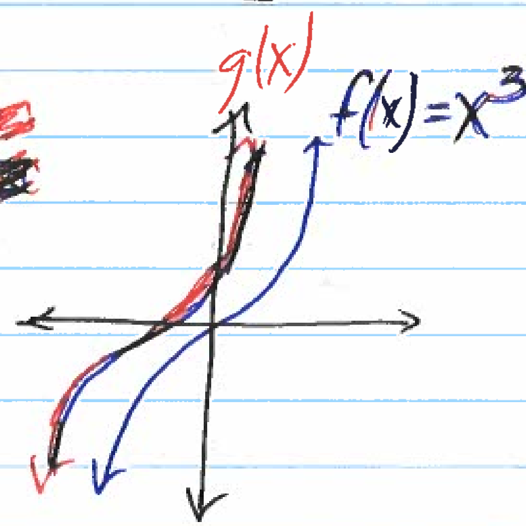
<u>Coefficients</u>: constants of polynomial

<u>Leading term</u>: the first term of a polynomial

Graphing Transformation of Monomial Function

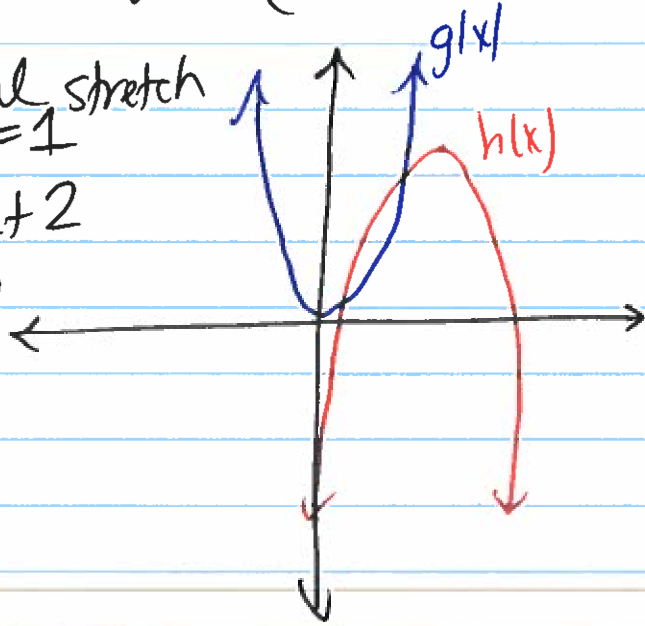
ie1 $g(x)=4(x+1)^3$

Vertical Stretch = 4
Shift 1 left



ie2 $h(x)=-(x-2)^4+5$ $g(x)=x^4$

- Vertical stretch = 1
- Right 2
- up 5

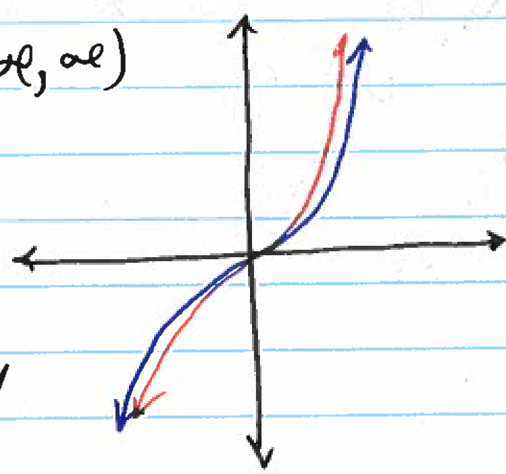


Graphing Combinations of Monomial Functions

graph, locate extremas and zero, explain related to parent monomial.

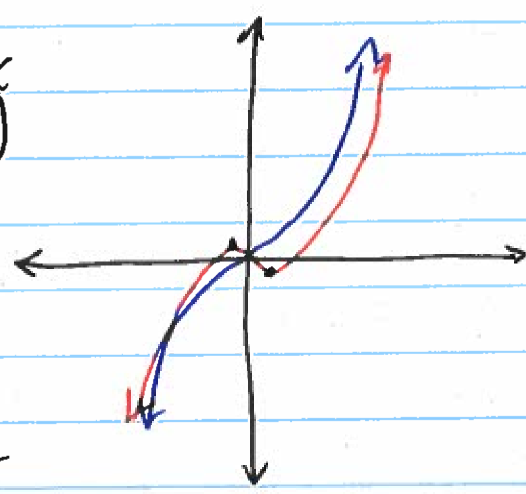
ie1 $f(x) = x^3 + x$ $g(x) = x^3$

- Increasing: $(-\infty, \infty)$
- No extremas
- One zero @ $(0,0)$
- Function: Odd



ie2 $g(x) = x^3 - x$ $h(x) = x^3$

- Local max $(-0.58, 0.38)$
- Local min $(0.58, -0.38)$
- Zeros at $x = -1, 0, 1$



- Odd function

End Behavior of Polynomial Functions

* Graph each function in the window

$[-5, 5]$ by $[-15, 15]$

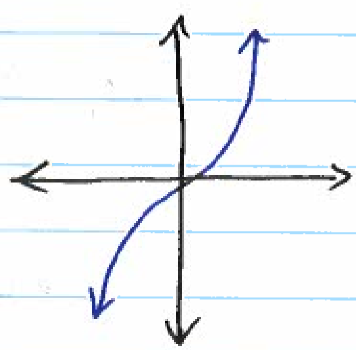
x-values $\quad$ y-values

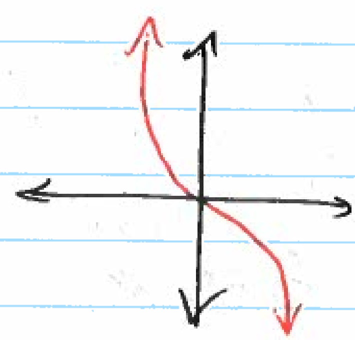
Describe end behaviors using

$\lim\limits_{x \to -\infty} f(x)$ & $\lim\limits_{x \to \infty} f(x)$

ie1 $f(x) = 2x^3$

ie2 $f(x) = -0.3x^5$





$\lim\limits_{x \to -\infty} 2x^3 = -\infty$

$\lim\limits_{x \to \infty} 2x^3 = +\infty$

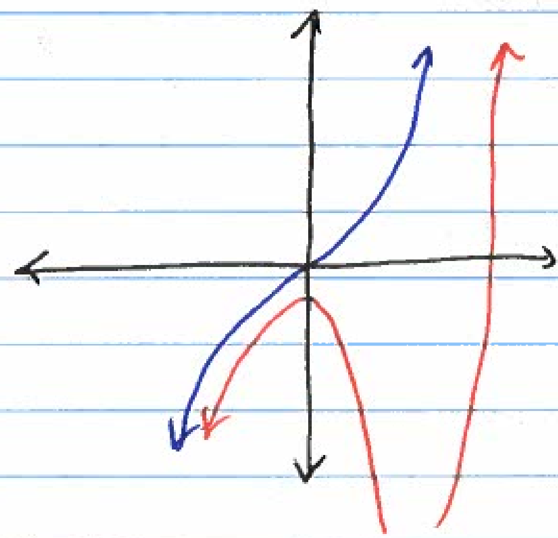
$\lim\limits_{x \to -\infty} -0.3x^5 = \infty$

$\lim\limits_{x \to \infty} -0.3x^5 = -\infty$

Comparing the Graphs of a Polynomial & its Leading Term

(ie 1) $f(x) = x^3 - 4x^2 - 5x - 3$ $\boxed{3}$

& $g(x) = x^3$ $\boxed{3}$



$\lim_{x \to -\infty} f(x) = \underline{-\infty}$

$\lim_{x \to \infty} f(x) = \underline{+\infty}$

$\lim_{x \to -\infty} g(x) = \underline{-\infty}$

$\lim_{x \to \infty} g(x) = \underline{+\infty}$

Applying Polynomial Theory

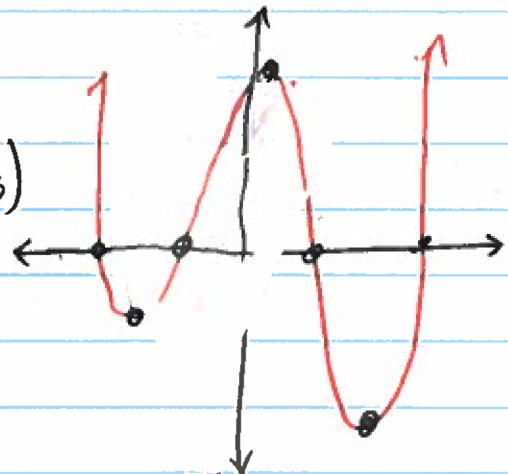
ie $g(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 35$

2ND graph min or max

Extremas: 3

min $(-2.5, -108)$; $(2.2, -43)$

max $(-0.4, 38.6)$



$[-5, 5]$ by $[-50, 50]$

x-values in window

y-values in window

2ND graph 2:zero

Zeros: 4

$(-3, 0)$; $(-2, 0)$

$(1, 0)$; $(3, 0)$

$\lim_{x \to -\infty} g(x) = +\infty$

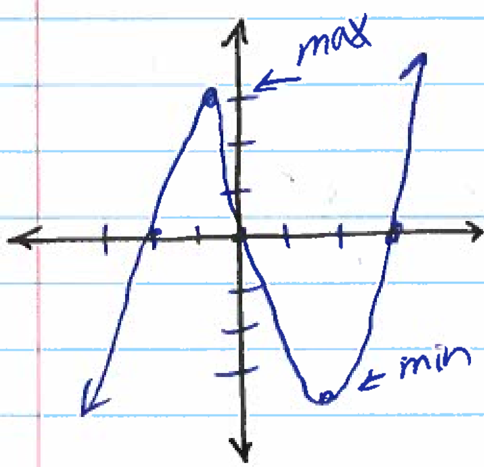
$\lim_{x \to +\infty} g(x) = +\infty$

Finding the Zeros of a Polynomial Function

ie 1 $f(x)=x^3-x^2-6x$; (X-intercept or where $y=0$)

* make sure you can do both!

Graph



Zeros: use calc

2nd calc 2: zeros

Zero $=(-2,0)$

Zero $=(0,0)$

Zero $=(3,0)$

Factor

$f(x) = x^3-x^2-6x$

☺ $= x^3-x^2-6x$

GCF $= x$ $\quad x(x^2-x-6)$

Factor $\quad x(x-3)(x+2)$

$-3x$

$+2x$

$-1x$

so $x(x-3)(x+2)=0$

$x=0$

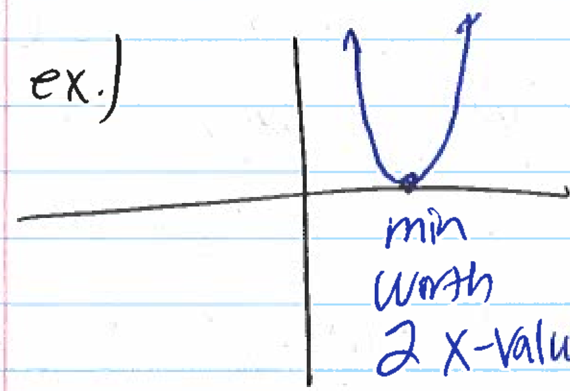
$x-3=0$
$+3 \quad +3$
$x=3$

$x+2=0$
$-2 \quad -2$
$x=-2$

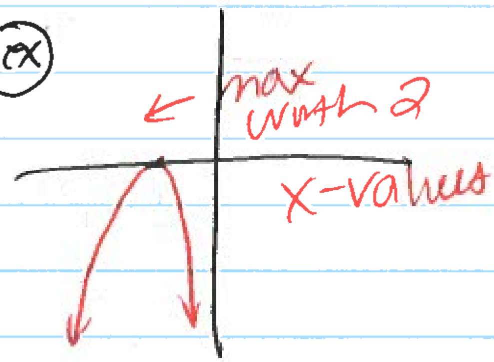
$(0,0)\ (-2,0)\ (3,0)$

Repeated Zero: When a graph touches the x-intercept at a max or min

ex.)

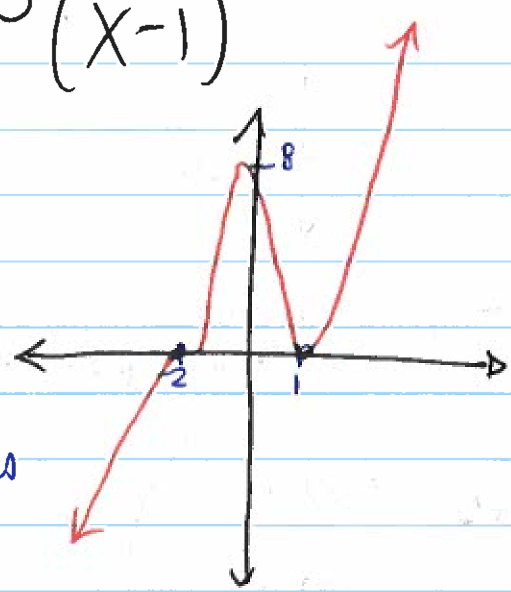


ex



ex 1 $f(x) = (x+2)^3(x-1)^2$

Degree: 5 (total)



X-int (zeros) $(-2,0)$ — 3 times

$(1,0)$ ← 2 times

End Behavior

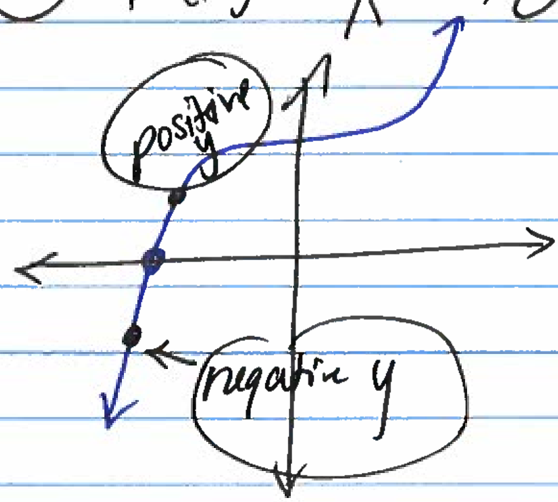
$\lim_{x \to -\infty} f(x) = \underline{-\infty}$ $\lim_{x \to \infty} f(x) = \underline{+\infty}$

Using Intermediate Value theorem

→ tells us that a sign change implies a real zero

(ie1) Why does a polynomial function of odd degree (power) have at least 1 real zero.

(ex) $f(x) = x^5 + 2$



2.4 Real Zeros of Polynomial Functions

Long Division of Polynomials:

used to determine missing factors of a polynomial.

Setup

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient and a remainder}$$

or dividend $\div$ divisor $=$ quotient

Using polynomial Long Division

to find Quotient and/or remainder if one exists ie 1 $2x^4-x^3-2$ divided by $2x^2+x+1$

Set up:

$$2x^2+x+1 \overline{)\,2x^4-x^3-2}$$

* Make sure No gaps in descending order of powers.

$2x^2+x^1+1$

this has no gaps

$2x^4-x^3-2$ gaps must fill in

$2x^4-x^3+\underline{0x^2+0x^1}-2$

these are filled in gaps in descending order

Now solve...

$$\begin{array}{r} x^2 \quad -1x \\ 2x^{\boxed{2}}+x+1 \overline{\big)\, 2x^4 - x^3 + 0x^{\boxed{2}} + 0x - 2} \\ + \left(-\cancel{2x^4} \mp 1x^3 \mp x^2\right) \quad \downarrow \\ \hline -\cancel{2}x^3 - x^2 + 0x \\ + \left(+2x^3 + x^2 + 1x\right) \\ \hline x - 2 \end{array}$$

remainder (only 2 terms not 3)

Step 1

$\frac{2x^4}{2x^2} = x^2$

line up

$\underline{x^2}$

Step 2:

mult

$x^2(2x^2+x+1)$

Step 3

Run (-) sign through all terms

Step 4

Simplify then bring down another term

Step 5

$\frac{-\cancel{2}x^3}{2x^2} = -1x$

Step 6

mult $-1x(2x^2+x+1)$

$-2x^3 - x^2 - 1x$

Step 7

Run (-) sign through all term 3

Solutions:

Remainder over the ~~with~~ original factor

$$\boxed{(2x^2+x+1)\left(x^2-x+\frac{x-2}{2x^2+x+1}\right)}$$

ie2 $\dfrac{3x^2+7x-20}{x-2}$ (← No missing terms)

$$
\begin{array}{r}
3x+13 \\
x-2\,\overline{)\,3x^2+7x-20} \\
+\left(-3x^2+6x\right) \\
\hline
13x-20 \\
+\left(-13x+26\right) \\
\hline
+6
\end{array}
$$

$$\boxed{(x-2)\left(3x+13+\frac{6}{x-2}\right)}$$

Step 1

$\dfrac{3x^2}{x}=3x$

Step 2

3x goes over 7x

Step 3

$3x(x-2)$

$3x^2-6x$

Step 4

Run (−) through

Step 5

$\dfrac{13x}{x}=13$

Step 6

13 goes over the 20

Step 7

$13(x-2)$

$13x-26$

Step 7

Run (−) through

Step 8

write Solution

5

ie3

$$3x - 5$$

$$x+4 \overline{)\, 3x^2 + 7x - 20}$$

← No terms missing

$$+ \left(^-3x^2 + ^-12x\right)$$

$$-5x - 20$$

$$+ \left(+5x + 20\right)$$

No remainder

$$\frac{3x^2}{x} = 3x$$

$$3x(x+4)$$

$$3x^2 + 12x$$

$$\frac{^-5x}{x} = -5$$

$$\boxed{(x+4)(3x-5)}$$

the factors

Synthetic Division:

- Works only with binomial divisor
- No missing terms still applies

$\boxed{\text{ie 1}}$ $$\frac{2x^3-3x^2-5x-12}{x-3}$$ ← No missing terms

add/subtract

add/subtract

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & \downarrow & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & 0 \end{array}$$

mult. mult. mult.

no remainder

Step 1
write all coefficients in order of degree

Step 2
$x-3=0$
$+3\ +3$
$x=3$
(outside value)

Step 3
drop 2 down

Step 4
mult.
$3\times2=6$
Bring up 6

Step 5
add/subtract

Step 6
$3\times3=9$
bring up

Step 7
mult. $3\times4=12$
bring up

Step 8
add/subtract

left blank

Step 9: Write solutions (factors)

$$\boxed{(x-3)(2x^2+3x+4)}$$

Start 1 degree less than original dividend

$\boxed{\text{ie2}}$ $\dfrac{5x^4-3x+1}{4-x}$ ← missing terms ← Reorder

$$\frac{5x^4+0x^3+0x^2-3x+1}{-x+4}$$

$$\begin{aligned} -x+4&=0\\ -4\;&\;-4\\ \hline \frac{-1x}{-1}&=\frac{-4}{-1}\\ x&=4 \end{aligned}$$

$$\begin{array}{r|rrrrr} 4 & 5 & 0 & 0 & -3 & 1 \\ & \downarrow & 20 & 80 & 320 & 1268 \\ \hline & 5 & 20 & 80 & 317 & 1269 \end{array}$$

mult

mult

$$A: (-x+4)\left(5x^3+20x^2+80x-317+\frac{1269}{-x+4}\right)$$

1 less degree than original

9

Rational Zeros Theorem:

→ Real zeros of polynomial functions

Irrational Zeros: irrational numbers

(ex) $f(x) = (4x^2 - 9)$

$= (2x+3)(2x-3)$

$\boxed{x = -\frac{3}{2} \quad x = \frac{3}{2}}$ Rational

(ex2) $f(x) = x^2 - 2$

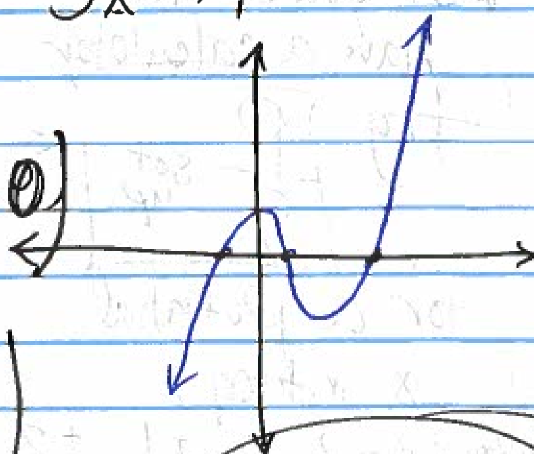
$(x+\sqrt{2})(x-\sqrt{2})$

$\boxed{x = -\sqrt{2} \quad x = \sqrt{2}}$ irrational

Finding the Rational Zeros

$\boxed{\text{ie1}}$ $f(x) = x^3 - 3x^2 + 1$

$x^{\text{int}} = (-0.5320889, 0)$ (irrational)



$x = (0.652, 0)$ irrational

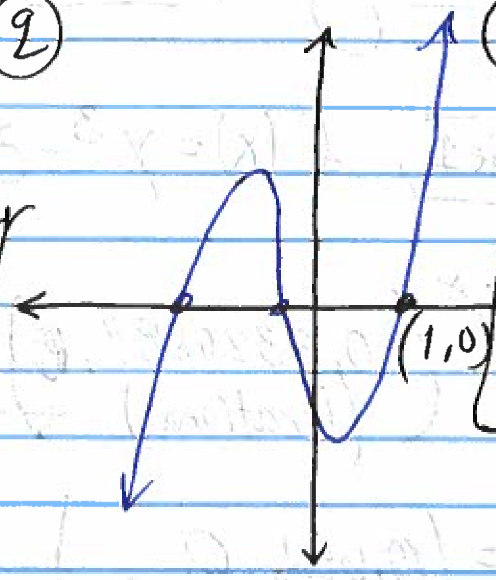
$x = (2.879, 0)$ irrational

all x-int are irrational #s

Fin.

$\boxed{\text{ie2}}$ $f(x) = 3x^3 + 4x^2 - 5x - 2$

q p



X-int
(1,0)
$(-\frac{1}{3}, 0)$
(−2,0)

* If you do not have a calculator try $\boxed{\frac{p}{q}}$ set up for a potential X-intercept

$$\frac{\text{factors of } -2}{\text{factors of } 3} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$$

then pick a value for synthetic division.

X-int

(1,0)

trying

$$\begin{array}{c|cccc} 1 & 3 & 4 & -5 & -2 \\ & \downarrow & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & 0 \end{array}$$

missing no term

so... X-int $(1,0)$ means $(x-1)$

$$=(x-1)(3x^2+7x+2)$$

write answers to synthetic division 1 lower degree

↓ factor

$$=(x-1)(3x+1)(1x+2)$$

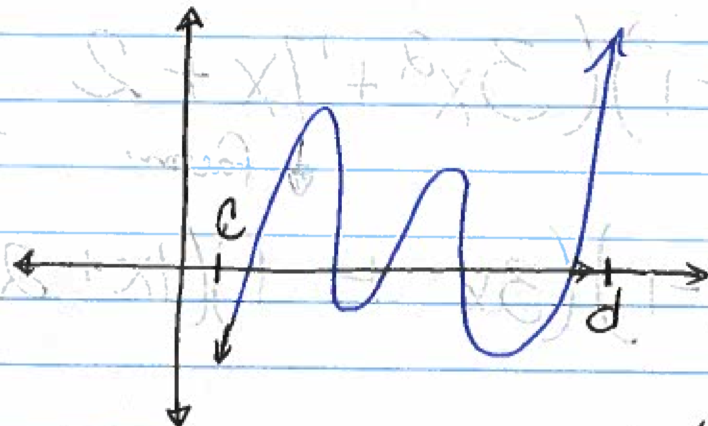
$x-1=0$ $\quad$ $3x+1=0$ $\quad$ $x+2=0$

$x=1$ $\quad$ $x=-\frac{1}{3}$ $\quad$ $x=-2$

$\boxed{(1,0),\ (-\tfrac{1}{3},0),\ (-2,0)}$

X-intercepts

Upper & Lower Bounds:



c is the lower bound (lower #)

d is the upper bound (upper/higher #)

Establishing Bounds for Real Zeros

← no terms missing

ie1 $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

in interval $[-2, 5]$

$$\begin{array}{r|rrrrr} -2 & 2 & -7 & -8 & 14 & 8 \\ & \downarrow & -4 & 22 & -28 & 28 \\ \hline & 2 & -11 & 14 & -14 & 36 \end{array}$$

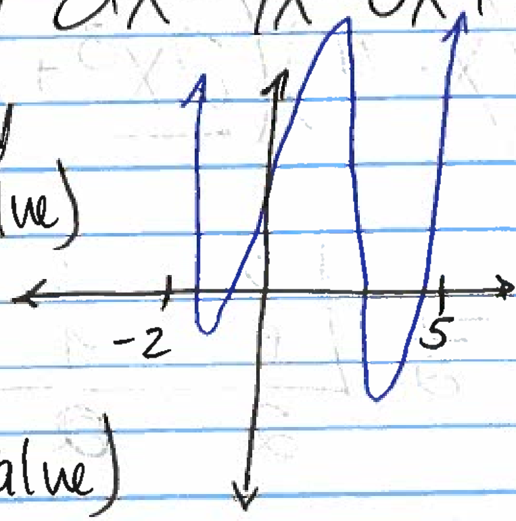
alt. signs
−2 lower bound

$$\begin{array}{r|rrrrr} 5 & 2 & -7 & -8 & 14 & 8 \\ & \downarrow & 10 & 15 & 35 & 245 \\ \hline & 2 & 3 & 7 & 49 & 253 \end{array}$$

all values positive

5 upper bound

(01) graph $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$



-2 lower bound (lowest X value)

5 upper bound (highest X value)

Finding the Real Zeros of a Polynomial Function (long hand)

& no term missing

(ie1) $f(x) = \underset{Q}{2}x^4 - 7x^3 - 8x^2 + 14x + \underset{P}{8}$

$\frac{P}{q} \quad \frac{8}{2} = \frac{\pm(1, 2, 4, 8)}{\pm(1, 2)} =$

Try 4

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & -8 & 14 & 8 \\ & \downarrow & 8 & 4 & -16 & -8 \\ \hline & 2 & 1 & -4 & -2 & ☺ \end{array}$$

4 works so...

$(x-4)(2x^3+x^2-4x-2)$

not simplified try another number

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & -4 & -2 \\ & \downarrow & -1 & 0 & 2 \\ \hline & 2 & 0 & -4 & 0 \end{array}$$

$-\frac{1}{2}$ works so...

$(x-4)(x+\frac{1}{2})(2x^2+0x-4)$

opposite of solution

$(\cancel{2x-2})(\cancel{2x+2})$ ~~factor~~

$2(x^2-2)$

$(x+\sqrt{2})(x-\sqrt{2})$

$2(x-4)(x+\frac{1}{2})(\cancel{2x-2})(\cancel{2x+2})$

$x-4=0$ $\quad$ $x+\frac{1}{2}=0$ $\quad$ $x+\sqrt{2}=0$ $\quad$ $x-\sqrt{2}=0$

$x=4$ $\quad$ $x=-\frac{1}{2}$ $\quad$ $x=\sqrt{2}$ $\quad$ $x=-\sqrt{2}$

$(4,0)$; $(-\frac{1}{2},0)$; $(\sqrt{2},0)$; $(-\sqrt{2},0)$