10.1 Limits : Motion : The Tangent Problem

P1 Computing Average Velocity

ie1 Auto travels 120 miles in 2 hrs : 30 min
What is it average speed velocity
Over the entire $2\frac{1}{2}$ hr. time interval?

$$\text{Ave Velocity } (V_{ave}) = \frac{\text{Change in position (miles) } \Delta s}{\text{Change in time (hours) } \Delta t}$$

$$V_{ave} = \frac{120 \text{ miles}}{2.5 \text{ hrs}} = 48 \text{ miles per hour}$$

Instantaneous Velocity

(Galileo used a ball rolling down a hill)

ic2 A ball rolls a distance of 16 feet in 4 seconds. What is the instaneous Velocity of the ball at a moment of time 3 seconds after the ball starts to roll?

(Ave. Velocity) $V_{ave} = \frac{\Delta s}{\Delta t} = \frac{16ft}{4sec} = 4$ feet per second

inst velocity $V_{ave} = \frac{\Delta s}{\Delta t} = \frac{0 ft}{0 \text{ second}} = \text{undefined}$

*answer will be approximated later in the chapter

Limits: a restriction or a point beyond something does not or may not pass.

P2 Using Limits to Avoid Zero Division

ie1 A ball rolls down a ramp so that its distance (s) from the top of ramp after (t) second is exactly t^2 feet. What is its instantaneous velocity after 3 seconds?

try interval [3, 3.1]

$$\frac{\Delta s}{\Delta t} = \frac{(3.1)^2 - 3^2}{3.1 - 3} = \frac{0.61}{0.1} = 6.1 \text{ ft per second}$$

try interval [3, 3.05)

$$\frac{\Delta s}{\Delta t} = \frac{(3.05)^2 - 3^2}{3.05 - 3} = \frac{0.3025}{0.05} = 6.05 \text{ ft per second}$$

Instantaneous Velocity
must be 6feet per second

FORMULA: interval $[3, t]$ as t approaches 3.

$$\lim_{t \to 3} \frac{\Delta s}{\Delta t} = \lim_{t \to 3} \frac{t^2 - 3^2}{t - 3}$$

$$= \lim_{t \to 3} \frac{(t+3)(t-3)}{(t-3)}$$ factored

$$= \lim_{t \to 3} (t+3) \cdot \left(\frac{t-3}{t-3}\right)$$

$$= \lim_{t \to 3} (t+3)$$

$$= \lim_{t \to 3} (3+3)$$

$$= 6$$

Definition (INFORMAL) Limit at $\underline{\underline{a}}$

$$\text{"} \lim_{x \to a} f(x) = L \text{"}$$

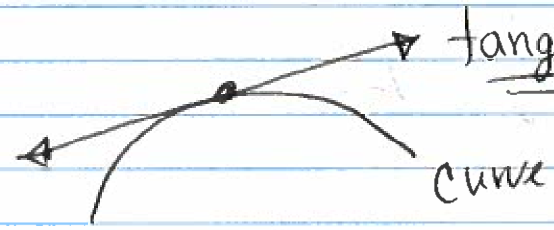
$f(x) \rightarrow$ gets arbitrarily close to L as x gets arbitrarily close (but not equal to) $\underline{\underline{a}}$.

The Connection to Tangent Lines

<u>FORMULA</u>: rolling ball down a ramp was that distance was proportional to the square of the elapsed time.

$$S = t^2$$

Tangent Lines: $\Longrightarrow$ the line drawn to meet a curve at 1 point



P3 Finding the Slope of a Tangent Line

ie1 Use limits to find the slope of the tangent line to the graph of $s = t^2$ at the point $(1,1)$

$$\lim_{t\to 1} \frac{\Delta s}{\Delta t} = \lim_{t\to 1} \frac{t^2 - 1^2}{t-1}$$

$$= \lim_{t\to 1} \frac{(t-1)(t+1)}{(t-1)}$$

$$= \lim_{t\to 1} (t+1)$$

$$= 1 + 1 = 2$$

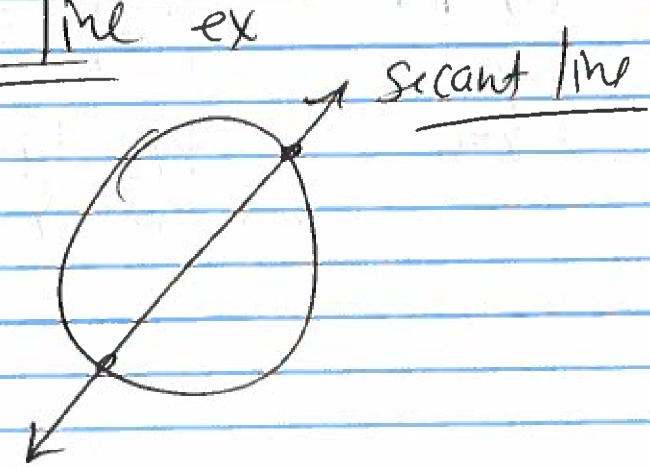
* The Derivative:

The rate of change (slope) of a function at a specific X value.
The slope of the straight line.
The slope of the tangent line to a curve.

Average Rate of Change (slope of secant line)

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

Secant line ex



Derivative at a point

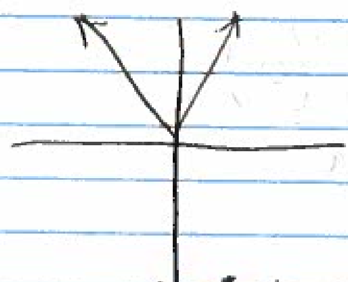
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ tangent line

or

other formula

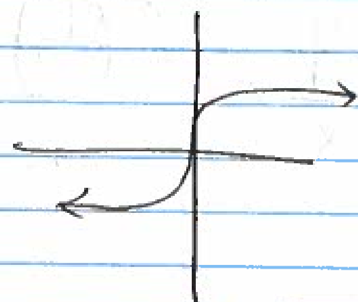
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ most used

ex With no definable slope



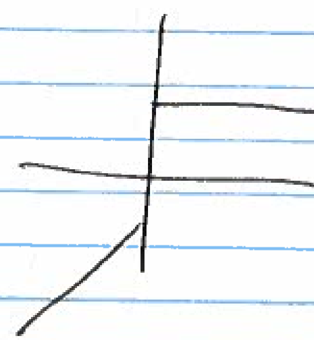
$f(x) = |x|$

no slope at $x=0$



$f(x) = \sqrt[3]{x}$

no slope at $x=0$



no slope b/c $x \neq 0$

P4 Finding Derivative at a Point

ie1 Find $f'(4)$ if $f(x) = 2x^2 - 3$

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h \to 0} \frac{2(4+h)^2 - 3 - \boxed{(2(4)^2 - 3)}}{h}$$

$$= \lim_{h \to 0} \frac{2(16 + 8h + h^2) - 32}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2h+16+} \; 32 + 16h + 2h^2 - 32}{h}$$

$$\lim_{h \to 0} = \frac{16h + 2h^2}{h}$$

$$\lim_{h \to 0} = \frac{\cancel{h}(16 + 2h)}{\cancel{h}}$$

$$\lim_{h \to 0} = 16 + 2(0) = 16$$

Derivative of a function f with respect to x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\frac{\Delta y}{\Delta x} \xRightarrow{\text{implies}} \left(\frac{dy}{dx} \Leftarrow \text{notation for derivative} \right)$

(P5) Finding the Derivative of a Function

a) $f'(x)$ if $f(x) = x^2$

b) $\frac{dy}{dx}$ if $y = \frac{1}{x}$

using $f(x) = \underline{\underline{x^2}}$

a) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \cancel{f}\, \frac{(x+h)^2 - (x)^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$\lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$\lim_{h \to 0} 2x + h$$

$$\lim_{h \to 0} = 2x + 0$$

$$= 2x$$

11

using $y = \frac{1}{x}$, x^{-1}

$= -1x^{-1-1}$

b) $f'(x) = 2x$

$= -1x^{-2}$

$= \frac{-1}{x^2}$

FORMULA

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{(x+h)} - \frac{1}{(x)}}{h}$$

$$\lim_{h \to 0} = \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \frac{\frac{x - x - h}{x(x+h)}}{h}$$

$$\lim_{h \to 0} = \frac{-h}{x(x+h)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0} = \frac{-1}{x(x+h)}$$

$$\lim_{h \to 0} = \frac{-1}{x(x+0)} = \frac{-1}{x^2}$$

Derivative Shortcut

by pass long formula

ie1 $3x^2+2$

$$\frac{dy}{dx} = 2(3)x^{2-1}$$

$$= 6x^1$$

ie2 x^2-2x^1+5

$$\frac{dy}{dx} = 2x^{2-1} - 2(1)\cancel{x^{1-1}} \rightarrow = x^0 = 1$$

or drops off

$2x-2$ when $x=3$

$$2(3)-2 = 6-2 = 4$$

very important * Derivative / tangent Line formula

ie3 $\frac{1}{2}x^{2} + 2x - 5$ when $x = 2$

$\frac{dy}{dx} = 2\cdot\frac{1}{2}x^{2-1} + 2(1)\cancel{x^{1-1}} = x^0 = 1$ drops off

$1x^1 + 2$

$x = 2$

$1(2) + 2 = 4$

$y = \frac{1}{2}(2^2) + 2(2) - 5$

$= 2 + 4 - 5$

$y = 1$

pt.-slope formula:

$y - y_1 = m(x - x_1)$

$= y - 1 = 4(x - 2)$

tangent Lin

ie4 $x^3 - 6x^2 + 12x - 9$ at $x = 0$

$\frac{dy}{dx} = 3x^{3-1} - 2(6)x^{2-1} + 12(1)x^{1-1}$

$3x^2 - 12x + 12$

$x = 0$

$f(0) = 0^3 - 6(0)^2 + 12(0) - 9$

$x = 0$

$y = 9$

$3(0)^2 - 12(0) + 12$

$= 12$

tangent line

$y - 9 = 12(m - 0)$

10.2 Limits & Motion: The Area Problem

"Distance equals rate x time"

$$D = R \times T$$

P1 Computing Distance Traveled

ie1 Car travels constant rate of 48 mph for 2 hours & 30 minutes. How does car travel?

$$D = R \times T$$

$$d = (48\ mi/hr)(2.5hr)$$

$$\boxed{d = 120\ \text{miles}}$$

10.1 set up ⇒ $\Delta S = 48\ mph\ \Delta t$

or $\dfrac{\Delta s}{\Delta t} = 48\ mph$

P2 Computing distance traveled

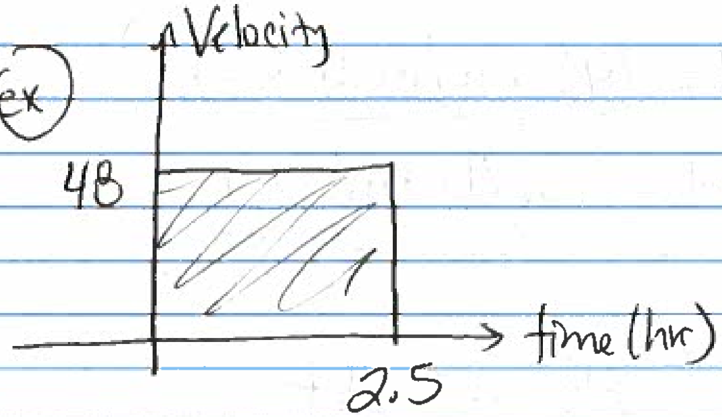
1. Ave rate of travel of 48 mi/hr for 2 hrs 30 min.

How does the auto travel?

$$\Delta s = \frac{\Delta s}{\Delta t} \cdot \Delta t = (48\text{ mph})(2.5\text{ hrs})$$
$$= 120 \text{ miles}$$

The Connection to Areas

ex

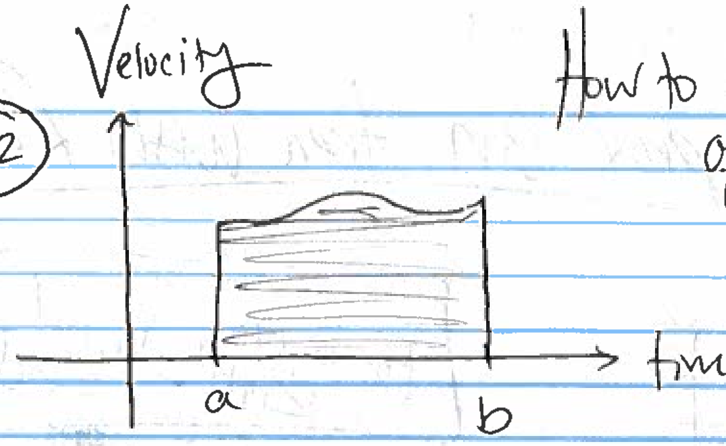


48×2.5

$= 120$ miles

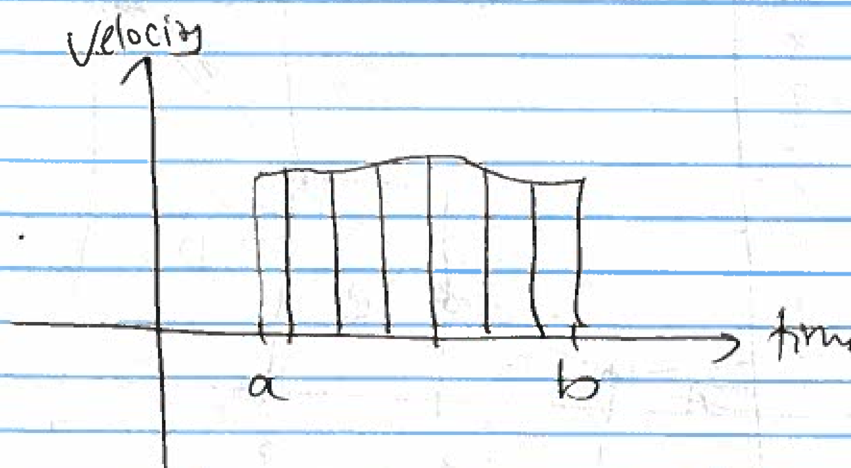
$A = b \cdot h$

ex 2



How to find area of a shape that is curved?

think of as...



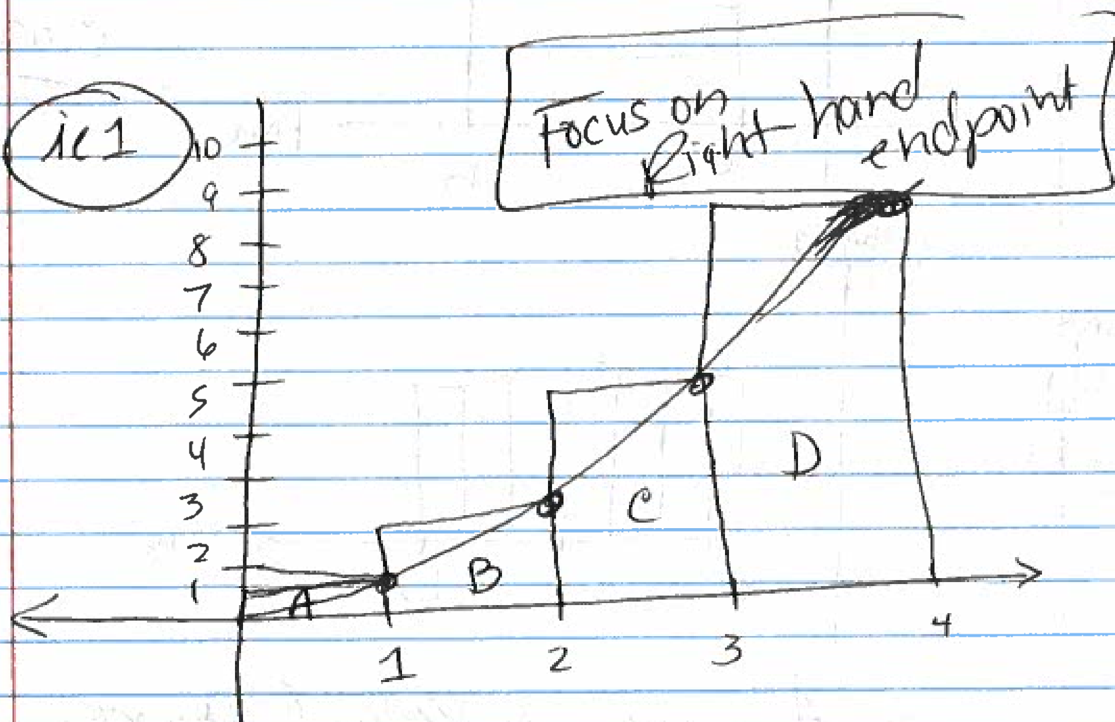
* A series of vertical strips. If narrow enough they form rectangles. The sum of those rectangles will give the total area.

~~Called Integrals~~

called Riemann's Sum

P3 Approx. an Area with Rectangles

ic1



Step 1: Create rectangles

Step 2: Find Area of each rectangle

Area A = $b \times h$
$1 \cdot 0.5$
0.5

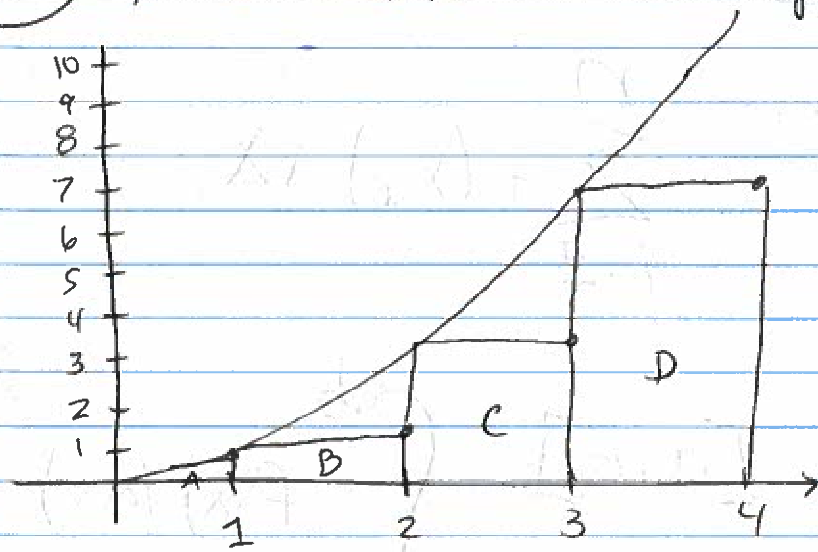
Area B = $b \times h$
1×2
$= 2$

Area C = $b \times h$
1×5
5

Area D: $b \times h$
1×9
$= 9$

Sum of all areas $= 0.5 + 2 + 5 + 9 = 16.5 \text{ units}^2$

ie2 Focus on left-hand endpoint



Area A $= b \times h$
$1 \cdot 0.5$
$= 0.5$

Area B
1×1
$= 1$

Area C $= 1 \times 3$
$= 3$

Area D
1×7
$= 7$

Sum for area $= 0.5 + 1 + 3 + 7$
$= 11.5 \text{ unit}^2$

The Definite Integral

formula

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

for real people $[a,b]$

$$\int_a^b f(x)(dx)$$

(e')

$$\int_{a \text{ - lower limit}}^{b \text{ - upper limit}} f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Best Formula

$$\int_{a \text{ - lower limit}}^{b \text{ - upper limit}} f(x)\,dx$$

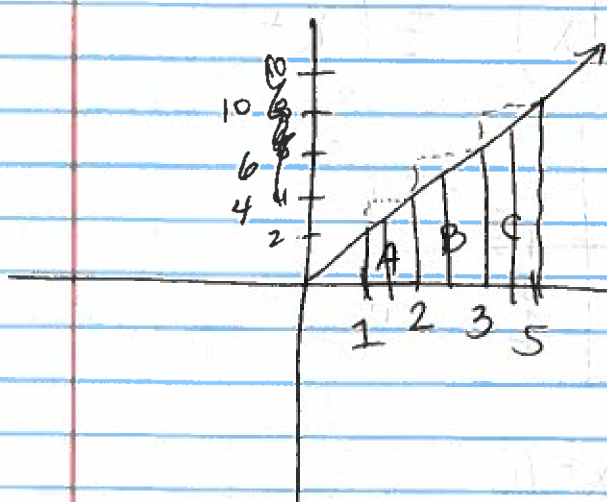
derivative of x

P4 Computing an Integral

ie1 Find $\int_1^5 2x\,dx$

Solution: ① equation is $f(x) = 2x$

② Graph it



③ Find area of shape between 1 & 5 on x-axis

on calc

3a) math 9: f(int) then type in equation

$(2x, x, 1, 3)$

$= 24 \text{ units}^2$

By hand

$A = 1 \times 4 = 4$

$B = 1 \times 6 = 6$

$C = 1 \times 10 = 10$

~~6+3x2~~

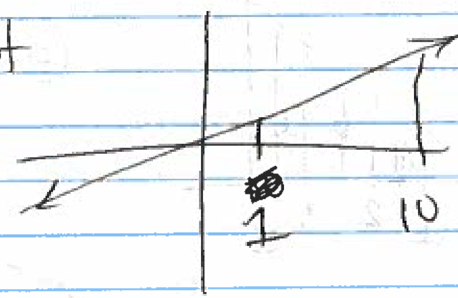
$4+6+10 = 20 \text{ units}^2$

⑦

ic2 $\displaystyle\sum_{K=1}^{10} \frac{1}{2}(K+1)$

Read as $\displaystyle\int_{1}^{10} \frac{1}{2}(X+1)$

1. Equation = $f(x) = \frac{1}{2}(x+1)$
2. Graph it



3. Finding area where $X=1$ to where $X=10$

4. calc

math 9: $fnInt\left(\frac{1}{2}(x+1), X, 1, 10\right)$

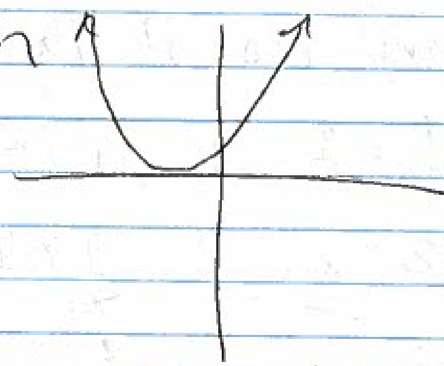
equal

29.25 units^2

ie3 $\sum_{K=1}^{10} \frac{1}{2}(K+1)^2$

① $f(x) = \frac{1}{2}(x+1)^2$

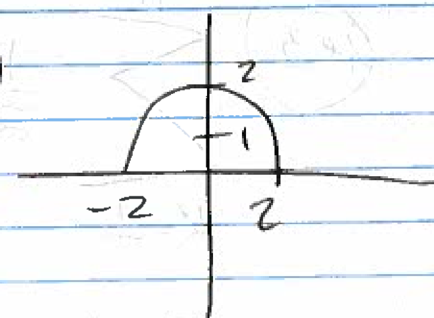
② graph



③ Math 9: fn(Int)$\left(\frac{1}{2}(x+1)^{\wedge 2}\right), x, 1, 10)$

$\boxed{= 220.5 \text{ units}^2}$

ie 4 $\int_{-2}^{2} \sqrt{4-x^2}\,(dx)$



calc. says Non-real #

So must do it by hand

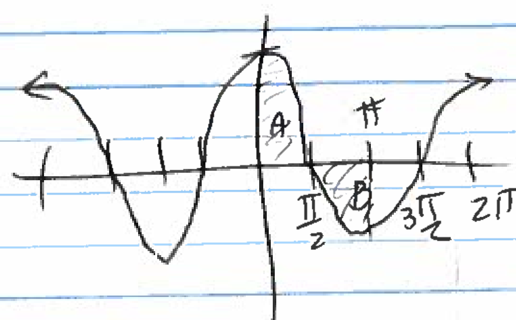
$A = 1 \times 1 = 1$ $\quad C = 1 \times 2 = 2$

$B = 1 \times 2 = 2$ $\quad D = 1 \times 1 = 1$

$\approx 6\text{units}^2$

or $2\pi u^2$

ie5 $\int_{0}^{\pi} \sin x\,dx$



graph by hand three

$A = 2$ unit

$A = 1 \times 1 = 1$

$B = 1 \times 1 = 1$

$1 + 1 = 2$

10.3 More on Limits

Definition (Informal) Limit at a

$$\lim_{x \to a} f(x) = L$$

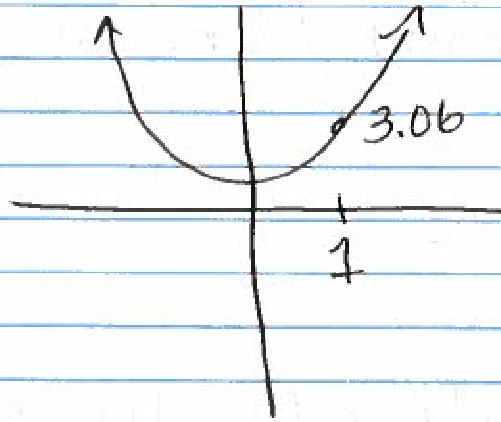
"f(x) gets arbitrarily close to L as x gets arbitrarily close but not equal to a"

$\boxed{P1}$ Finding a Limit

ie1 Find $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

3 methods to find the limit next page ⟹

① Solve Graphically ⇒ $\lim_{X \to 1} \left(\frac{X^3-1}{X-1}\right)$



$X = 1.02$

then $\boxed{y = 3.06}$

② Solve Numerically table after graphing

	X	y
above	.999	2.997
	1	Error
below	1.001	3.003

← will show error because 1 is the limit

$$\begin{array}{r} 3.003 \\ +\ 2.997 \\ \hline 6.000 \end{array}$$

$2\overline{)\cancel{6}} = 3$

$y = 3$

③ Solve Algebraically (Factor to simplify)

$$\lim_{x \to 1} \left(\frac{x^3-1}{x-1}\right)$$

diff of cubes

$$\frac{(x^3-1)}{(x-1)} = \frac{\cancel{(x-1)}(x^2+1x+1)}{\cancel{(x-1)}}$$

tip: usually cancels out w/denominator

$$\lim_{x \to 1} (x^2+1x+1)$$

$$= (1^2+1(1)+1)$$

$$= 1+1+1$$

$$y = 3$$

Left Blank

Properties of Limits

If $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$

1. **Sum Rule:**

$$\lim_{x\to c} f(x) + g(x) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$$

2. **Difference Rule**

$$\lim_{x\to c} f(x) + g(x) = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$$

3. **Product Rule**

$$\lim_{x\to c} (f(\boxed{x}) \cdot g(x)) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$

4. **Constant Multiple Rule**

$$\lim_{x\to c} (K \cdot g(x)) = K \cdot \lim_{x\to c} g(x)$$

5. **Quotient Rule:**

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} (f(x))}{\lim_{x \to c} g(x)}$$

$$\lim_{x \to c} g(x) \neq 0$$

6. **Power Rule**

$$\lim_{x \to c} (f(x))^n = \left(\lim_{x \to c} f(x)\right)^n$$

7. **Root Rule**

$$\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} \qquad n \geq 2$$

* must be real #'s

P2 Using Limit Properties

Given: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Find the following limits.

ie1 $\lim_{x \to 0} \frac{(x + \sin x)}{x}$

$= \lim_{x \to 0} \left(\frac{x}{x} + \frac{\sin x}{x} \right) \Leftarrow$ Sum Rule

$= \lim_{x \to 0} (1 + 1)$

$= 2$

ie2 $\lim_{x \to 0} \frac{1-\cos^2 x}{x^2}$

$\lim_{x \to 0} \frac{\sin^2 x}{x^2}$ ← pythag. idutity

$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)$ product rule

$= 1 \cdot 1$

$= 1$

ie3 $\lim_{x \to 0} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{x}}$

$= \lim_{x \to 0} \sqrt[3]{\frac{\sin x}{x}}$ combined term

Root Rule

$= \sqrt[3]{\lim_{x \to 0} \left(\frac{\sin x}{x}\right)}$

$= \sqrt[3]{(1)}$

$= 1$

Limits of Continuous Functions

Find the limits

$\boxed{\text{ie1}}$ $\lim\limits_{x \to 0} \dfrac{e^x - \tan x}{\cos^2 x}$

$$= \frac{\lim\limits_{x \to 0} (e^x - \tan x)}{\lim\limits_{x \to 0} \cos^2 x} \quad \text{Quotient Rule}$$

$$= \frac{\lim\limits_{x \to 0} e^0 - \cancel{\lim\limits_{x \to 0}} \tan 0}{\lim\limits_{x \to 0} (\cos 0)^2} \quad \begin{matrix} \leftarrow \text{diff}^{n} \\ \leftarrow \text{power Rule} \end{matrix}$$

$$= \frac{1 - 0}{1^2} = 1$$

$\boxed{ie2}$ $\lim_{n\to 16} \frac{\sqrt{n}}{\log_2 n}$

$= \frac{\lim_{n\to 16} \sqrt{n}}{\lim_{n\to 16} \log_2 n}$ ⇐ Quotient Rule

$= \frac{\sqrt{16}}{\log_2 16}$ ⇐ limits of continuous function

$= \frac{\sqrt{16}}{\frac{\log 2}{\log 16}}$ or

or

$2^x = 16$

$2^x = 2^4$

$x = 4$

$= \frac{4}{.25} = \frac{1.4}{4}$

$= 1$

day 2

One-sided & 2 Sided Limits

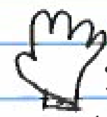
Left hand: $\lim_{x \to c^-} f(x)$ — the limit of f(x) as it approach c from the left.

Visual from left →

Slap from Left-right to right

Right hand: $\lim_{x \to c^+} f(x)$ — the limit of f(x) as it approaches c from the right

Visual from right ←



Slap from right to your left

$\boxed{P4}$ Finding Left and Right hand limits ie1 Find $\lim_{x \to 2^-}$ & $\lim_{x \to 2^+}$ of $f(x)$ where $f(x) = \begin{cases} -x^2+4x-1 & x \le 2 \\ 2x-3 & \boxed{x > 2} \end{cases}$

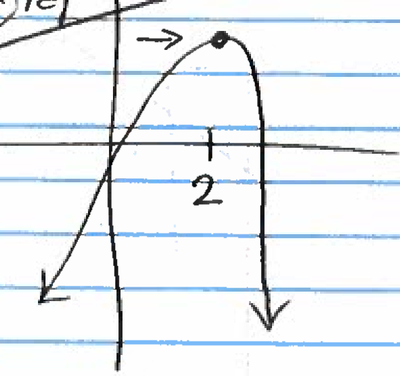
left side

right side

Left Limit

$$\lim_{x \to 2^-} -x^2+4x-1$$

Graph



table

x	y
2	3

$y=3$

Algebra

or $\lim_{x \to 2^-} -x^2+4x-1$

$-(2)^2+4(2)-1$

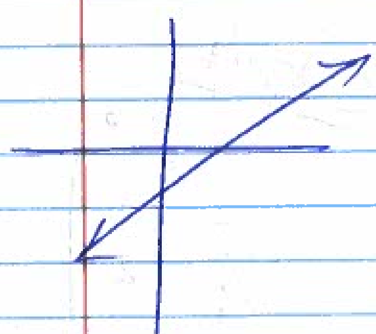
$-4+8-1$

$4-1 = 3$

Right side limit

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x-3)$$

Graph



Table

x	y
2	1

Algebra

$2(2)-3$

$= 4-3 = 1$

Two-sided limit

→ only has 2 sided limit
if left-side limit & right-side limit are equal.

day3 ~~#23-41~~ #41-

P5 Finding a Limit at a Point of Discontinuity (Hole or break in a line or curve)

$$\text{Let } f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$$

Find $\lim_{x\to 3} f(x)$ & prove that f is discontinuous at $x=3$

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$$

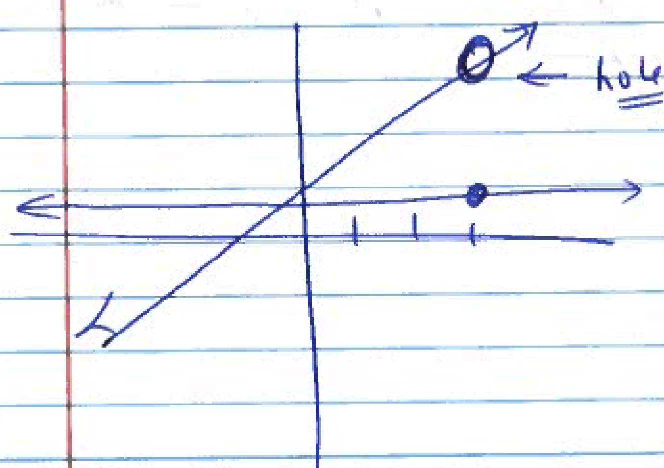
$$= \lim_{x\to 3} (x+3) = 3+3 = \underline{\underline{6}}$$

$(3,2)$ not same & $(3,6)$

Because $f(3) = 2 \neq \lim_{x\to 3} f(x)$

&

discontinuous at $x=3$



P6 Finding 1 sided & 2 sided Limits

Let $f(x) = \text{int}(x)$ (the greatest integer function)

also called step function

equal or less than X

Find:

ie1 $\lim_{x \to 3^-} \text{int}(x)$

example

$\text{int}(3) = 3$

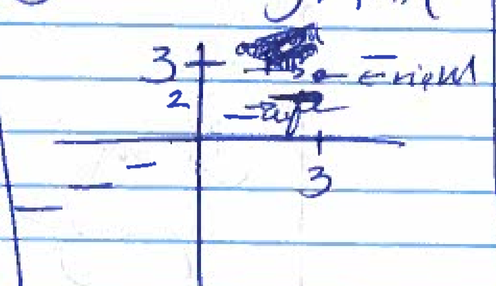
$\lim_{x \to 3^-} \text{int}(x) = 2$

has to have part of the graph to count

Graph

$y = \boxed{\text{math}}$ Num

5: int(



ie2 $\lim_{x \to 3^+} \text{int}(x)$

$\lim_{x \to 3^+} \text{int}(x) = 3$

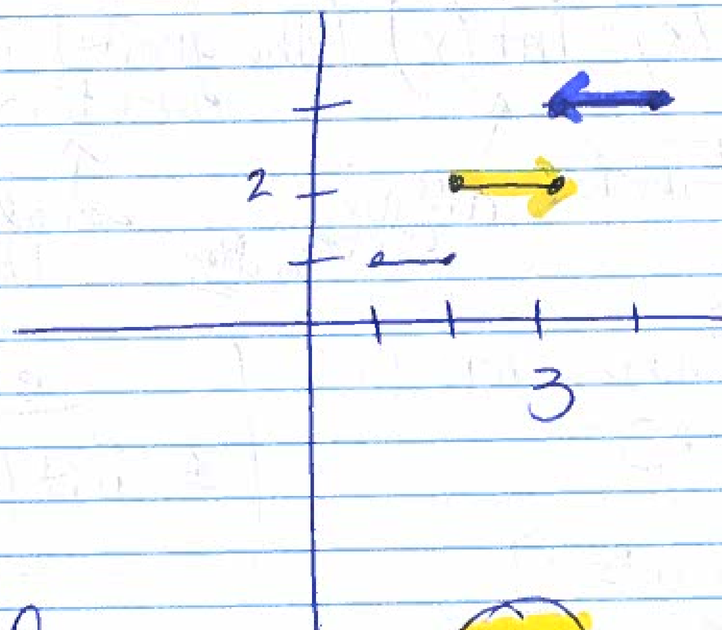
ie3 $\lim_{x \to 3} \text{int}(x)$ = does not exist

both must be the same

Better Visual on opposite side

17

$y = \boxed{\text{math}}\ 5{:}\ \text{int}(x)$



$$\lim_{x \to 3^-} \text{int}(x) = 2$$

* must have value from left to $x=3$

$$\lim_{x \to 3^+} \text{int}(x) = 3$$

* must have value from right to $x=3$

Limits Involving Infinity

$\lim\limits_{x \to \infty} f(x) = L$ (f has a limit L as x approaches ∞)

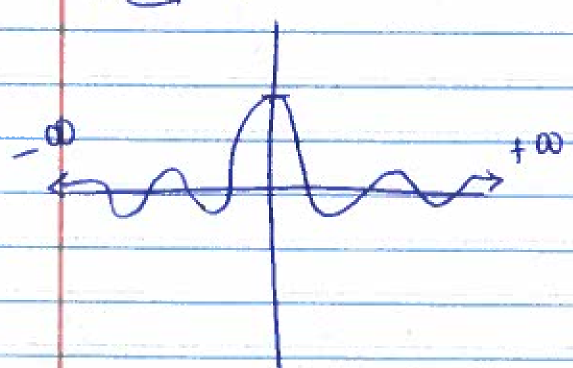
$\lim\limits_{x \to -\infty} f(x) = L$ (f has a limit L as x approaches $-\infty$)

$\boxed{P7}$ Investigating Limits as $x \to \pm\infty$

Let $f(x) = \frac{\sin x}{x}$. Find $\lim\limits_{x \to \infty} f(x)$ & $\lim\limits_{x \to -\infty} f(x)$

Must be in radians

1) graph on calc



$\lim\limits_{x \to \infty} f(x) = ☺$

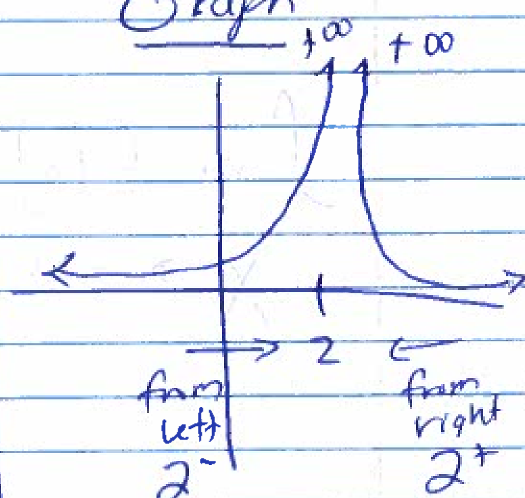
$\lim\limits_{x \to -\infty} f(x) = ☺$

P9 Unbounded Limits (No end)

$-\infty$ or ∞ Limits

Find $\lim_{x \to 2} \frac{1}{(x-2)^2}$

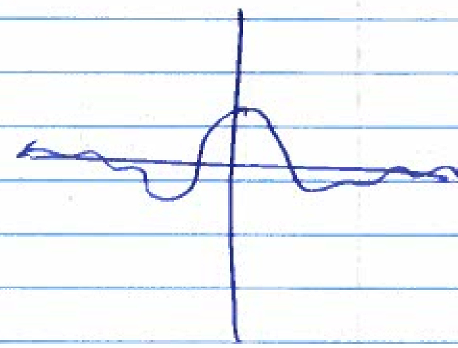
Graph



$$\boxed{\begin{aligned} \lim_{x \to 2^-} \frac{1}{(x-2)^2} &= +\infty \\ \lim_{x \to 2^+} \frac{1}{(x-2)^2} &= +\infty \end{aligned}}$$

P10 Limits at ☺

Find $\lim_{x \to} \frac{(\sin x)}{x}$



2nd Table set

Tblstart = .01

ΔTbl = .01

x	y
−.01	.999
0	error
.01	.999

add together
divid. by
2
.99 = 1

= 1

10.4 Numerical Derivatives & Integrals

Symmetric difference quotient for derivatives

$$\text{NDER} f(a) = \frac{f(a+h) - f(a-h)}{2h}$$

calculator derivative

NDER in ∩ Deriv. on our calc

$$\text{NDER} f(a) = \frac{f(a+0.001) - f(a-0.001)}{0.002}$$

Numerical derivative of f

P1 Computing a numerical derivative

ie1 $f(x) = x^3$. Compute NDER $f(2)$ by calculating the symmetric difference quotient with $h = 0.001$.

Aweful method!

$$\text{NDER } f(x) = \frac{f(2+0.001) - f(2-0.001)}{0.002}$$

$$\frac{f(2.001) - f(1.999)}{0.002}$$

$$\frac{(2.001)^3 - (1.999)^3}{0.002}$$

$$\frac{8.012 - 7.988}{0.002}$$

$$= 11.9$$

$$= 12.0$$

$f(x) = x^3$

More friendly Version:

$f'(2) = 3x^2$ ← derivative

Algebraically $f'(2) = 3(2^2)$

$= 3 \cdot 4$

$= 12$

or on calc.

$\boxed{\text{Math}}$ $\boxed{\text{8: Nderive}}$ $(x^3, x, 2$

$= 12.001$

another example

$f(x) = \sin 3x$ @ $x = \pi$

math 8: NDeriv $(\sin(3x), x, \pi)$

$= -3.00$

NINT = fnInt
on our calc

P2 Finding a Numerical Integral

To find area of the region ...

ie) $y = \frac{1}{x}$ from $x = 1$ to $x = 4$

2 values b/t

use $\boxed{\text{fnInt}\left(\frac{1}{x}, x, 1, 4\right)}$ calc. work

Set up to show work in college

$$\int_1^4 \frac{1}{x}\,(dx)$$

P3 Finding Distance Traveled

Auto driven @ variable rate along a test track for 2 hours so that its velocity at any time t $(0 \le t \le 2)$ is given by

$v(t) = 30 + 10\sin 6t$ miles per hour

How far does auto travel during 2 hour test? (~~net change problem~~)

$$\int_0^2 30 + 10\sin \cdot 6x \,(dx).$$

fN Int$(30 + 10\sin(6x), x, 0, 2)$

60.26 miles

do this one

ex $f(x) = |\cos x|, [0, \pi]$

$= 2$

17 in Hwk

5

Skip 2020-2021

P4 Finding Derivatives from Data

pg. 829 table intervals @ 0.04 seconds

a) estimate instantaneous speed of the ball @ $t = 0.2$ second

0.2 is between 0.16 ; 0.24 time

$0.16 \downarrow$ 5.45 speed

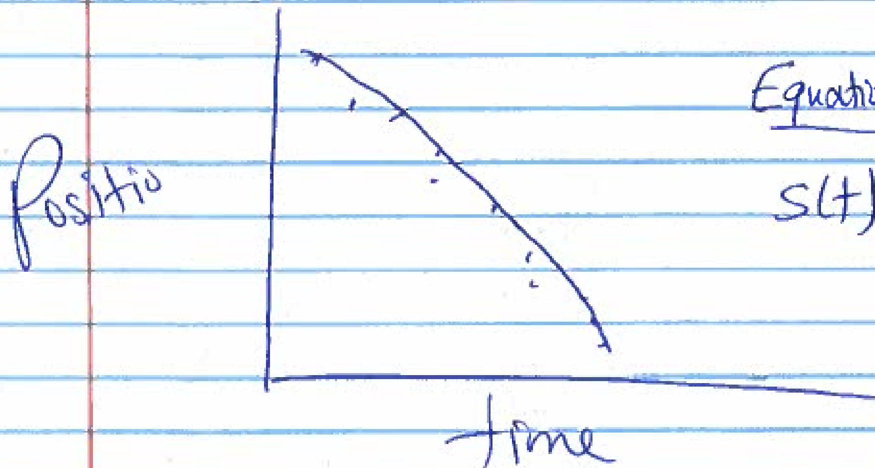
$0.24 \downarrow$ 4.30 speed

$$S'(0.2) = \frac{\Delta S}{\Delta t} = \frac{4.30 - 5.45}{0.24 - 0.16} = -14.375$$

$= \cancel{14.375}$

$= 14.375$ feet per second at $t = 2$

b) Draw the scatter plot & use quadratic regression (curved line)



<u>Equation</u>: given

$s(t) \approx -17.12t^2 - 7.74t + 7.13$

c) fn(Int$(-17.12\cancel{x}^2 - 7.74t + 7.13, x, 0, 2)$

$$\int_0^2 -17.12x^2 - 7.74x + 7.13, x, 0, 2) = -14.588$$

Close to answer in (a)

through #23

(7)

P5 Finding Derivatives from Data