1.1 Modeling & Equation Solving

Numerical Models – derived by scientists & engineers

→ Mathematical modeling: approx. for purpose of studying or predicting behaviors

* Thanks to advances in computer technology

→ Numerical model: numbers or data are analyzed to gain insights

* ex.: baseball standings
ex: global economy

Algebraic Models: uses formulas to relate to variable quantities

Ex 3 pg. 71

18" x 24" Rectangle
24" diameter Round

Area = 18 x 24

A = 432 in²

A = 432 in²

$A = \pi r^2$

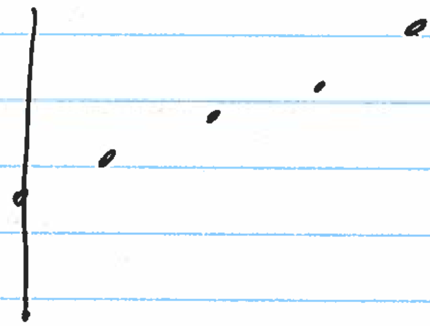
$A = \pi (24^2)$

$A = 452.4\ in^2$

Round pizza

Ex 5 pg. 73 Fitting a Curve to Data

Table ↳ (in Book)



t = years

F = % of females in prison

x	t	0	5	10	15	20
approx y	F	3.8	4.4	5.5	5.9	6.7

Find slope: $\dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{\Delta y}{\Delta x}$

y-int = 3.8 $\dfrac{4.4 - 3.8}{\cancel{3.8}\ \ 5-0} = \dfrac{0.6}{5} = 0.12$ approx due to graph

$y = mx + B$ $\boxed{y = 0.12x + 3.8}$

Solving an Equation with quad formula versus graphing

(ic) $X^2 = 10 - 4X \Rightarrow \underset{a}{X^2} + \underset{b}{4}X - \underset{c}{10} = 0$ ← make equal to zero

quad formula

$$X = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = 4$
$c = -10$

$$X = \frac{-4}{2(1)} \pm \frac{\sqrt{4^2 - 4(1)(-10)}}{2(1)}$$

$$X = -2 \pm \frac{\sqrt{16 + 40}}{2}$$

$$X = -2 \pm \frac{\sqrt{56}}{2}$$

$X = -2 + \frac{\sqrt{56}}{2}$

$X = 1.74$

$X = -5.75$

$X = -2 - \frac{\sqrt{56}}{2}$

graphing

same steps as previous problem

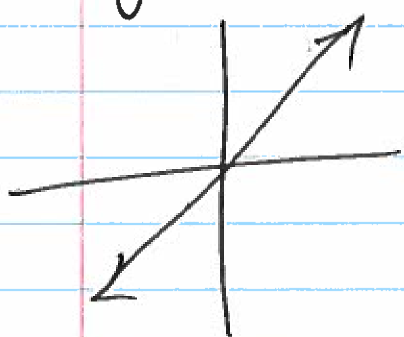
$X = 1.74$

$X = -5.75$

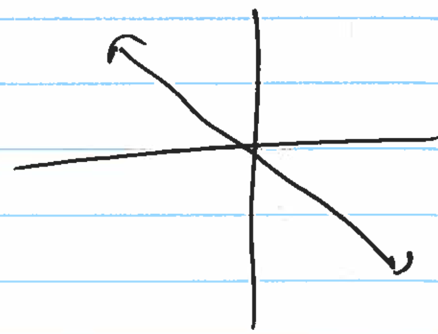
do #3 Through [illegible]

Graphs for your reference called parent graphs

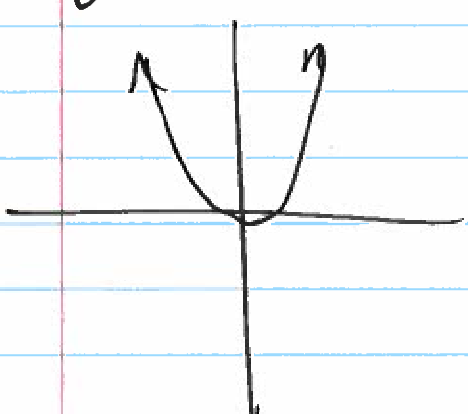
$y = x$



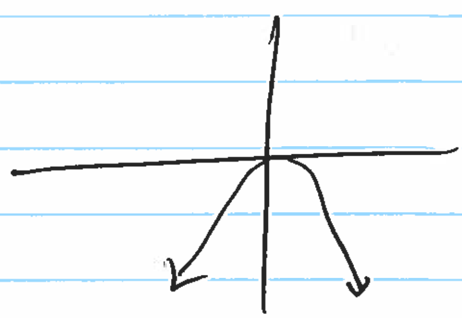
$y = -x$



$y = x^2$



$y = -x^2$



1.2 Functions & Their Properties

Function: where you have 1 x value for every 1 y value

$y = f(x)$

* x values cannot be used more than 1 time.

ex

x	y
-1	3
0	4
1	5
2	6
3	7

Function

ex2

x	y
-1	3
0	4
1	5
1	6
3	7

not a function

(cannot use x more than 1 time)

ex3

x	y
-1	4
0	3
1	2
2	2
3	6

function

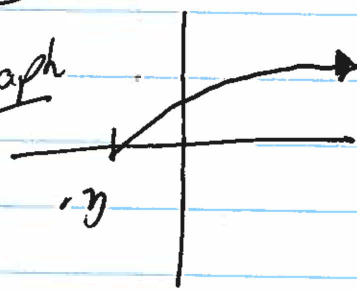
can use y more than 1 time

Finding Domain & Range:

U - union
2 sets combined

ie 1 $f(x) = \sqrt{x+3}$

① graph

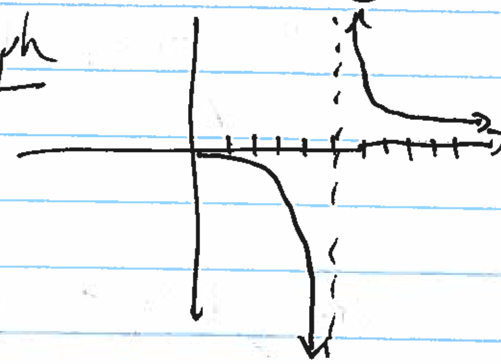


② D: $[-3, \infty)$

③ R: $[0, \infty)$

ie 2 $g(x) = \dfrac{\sqrt{x}}{x-5}$

① graph

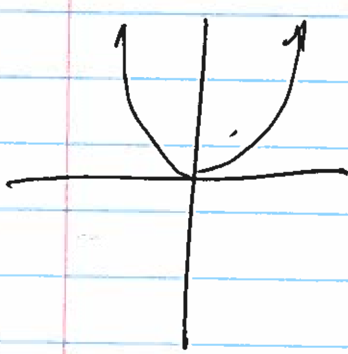


② D: $[0, 5) \cup (5, \infty)$

③ R: $(-\infty, 0] \cup (0, \infty)$

ie 3 $f(x) = \left(\dfrac{\sqrt{3}}{4}\right)x^2$

① graph

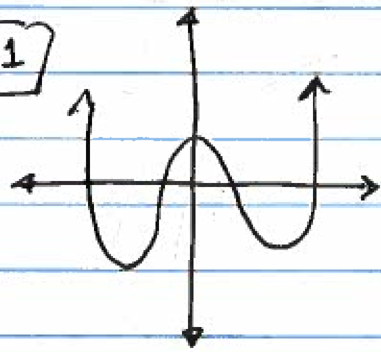


D: $(-\infty, \infty)$

R: $[0, \infty)$

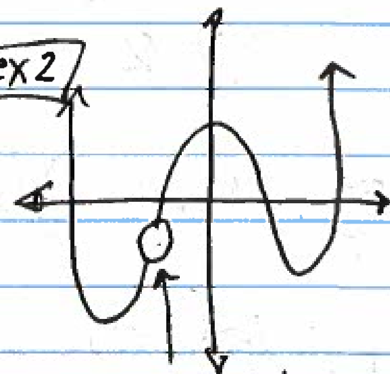
Continuity: Continuous or removeable discontinuous (piecewise functions)

ex 1



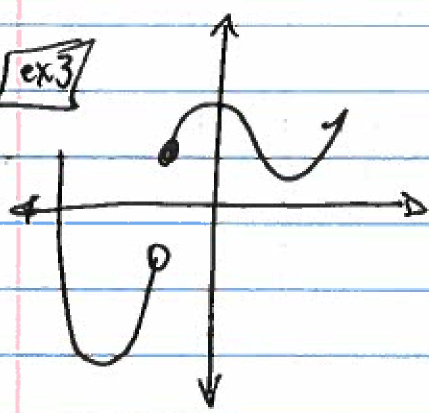
Continuous at all x values

ex 2



Removable discontinuity (missing value)

ex 3

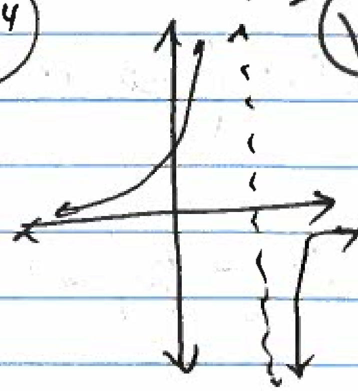


Jump discontinuity

ex 4

Vertical asymptote

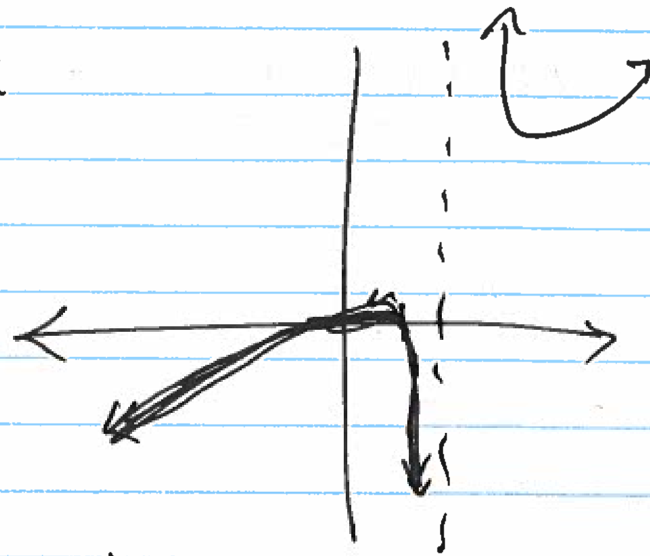
VA



Infinity discontinuity

$$f(x) = \frac{x^2 - x}{x - 2}$$

$x \neq 2$



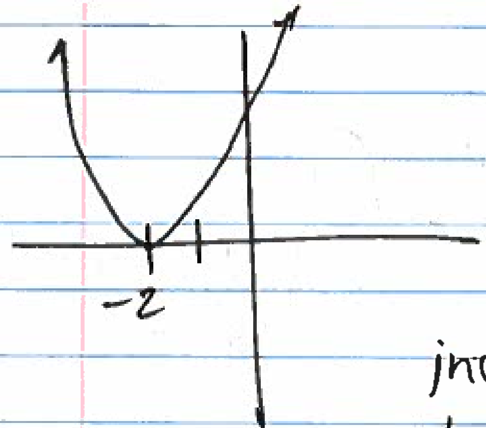
D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 0] \cup [6, \infty)$

Remember: (or) not included

[or] included...

ie 1 $f(x) = (x+2)^2$

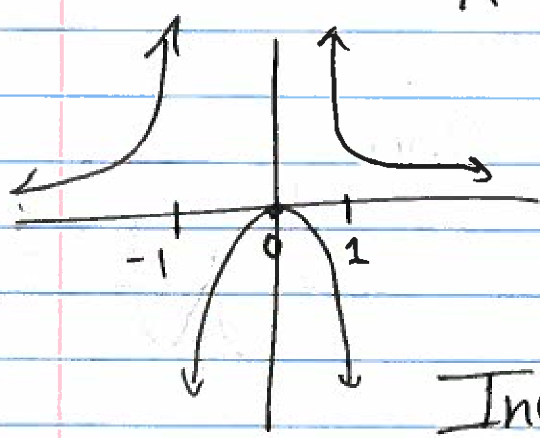


D: $(-\infty, \infty)$

R: $[0, \infty)$

increases: $[-2, \infty)$
decrease: $(-\infty, -2]$

ie 2 $g(x) = \dfrac{x^2}{x^2-1}$



D: $(-\infty, -1) \cup [0] \cup [2, \infty)$

R: $(-\infty, 0] \cup [2, \infty)$

Increasing: $(-\infty, -1)$;
$(-1, 0]$

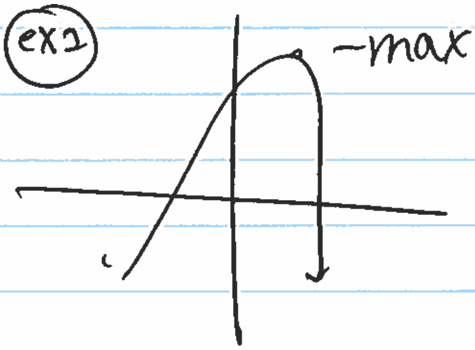
Decreasing: $[0, 1)$
; $(1, \infty)$

Local / absolute Extrema

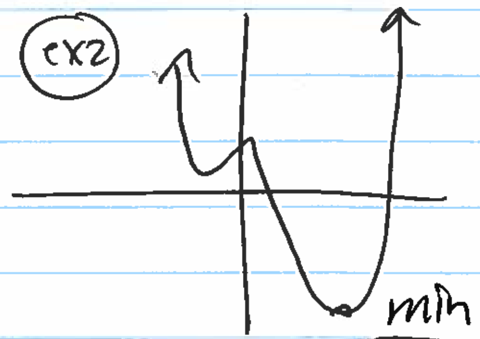
Local maximum: highest portion of graph

Local minimum: lowest portion of graph

ex1

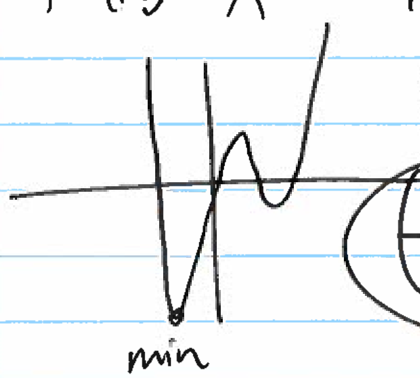


ex2



ie1 Find local min or max.

$f(x) = x^4 - 7x^2 + 6x$



min.

(-2, -24.06)

1. graph
2. 2nd calc
3. min
4. left (enter)
   right (enter)
   enter

solution -2, -24.06

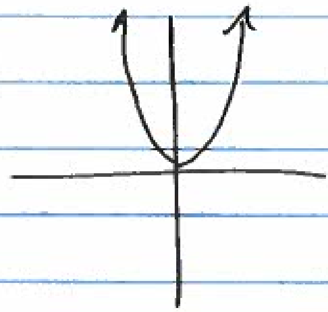
& end behaviors odd & even

Symmetry: looks like a mirror image with line of reflection. (line of symmetry)

even: arrows end in same direction

odd: arrow in opposite direction

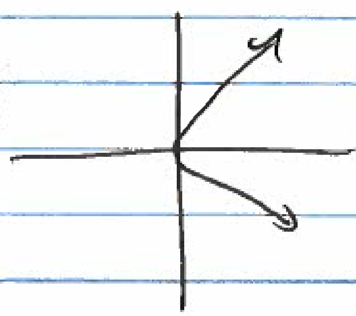
ex



L.S. = y-axis

even

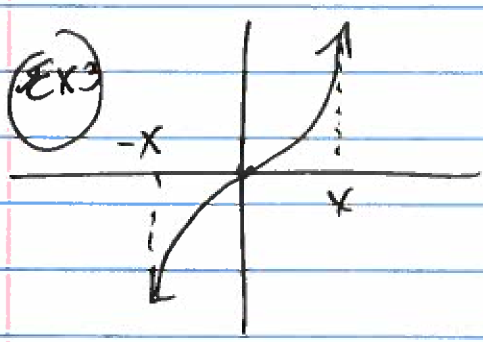
ex2



L.S = x-axis

even

Ex3



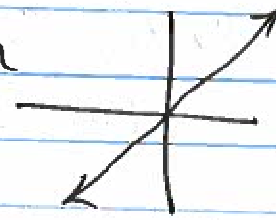
L.S. the origin

odd

1.3 12 Basic Functions

The Identity Function

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

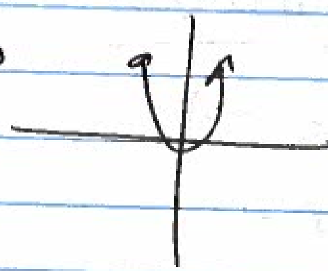


$f(x) = x$

The squaring function

Symmetric

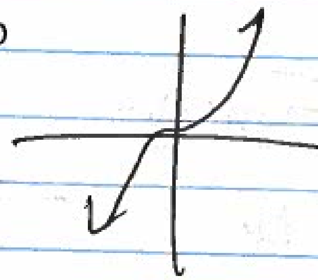
D: $(-\infty, \infty)$
R: $[0, \infty)$



$f(x) = x^2$

The cubing function

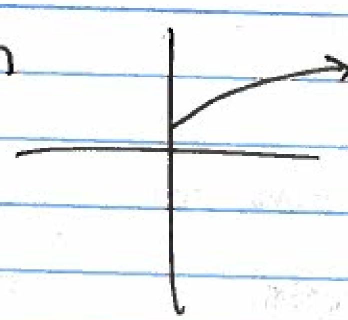
D: $(-8, \infty)$
R: $(-\infty, \infty)$



$f(x) = x^3$

Square Root Function

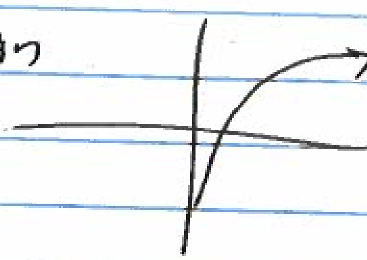
D: $[0, \infty)$
R: $[0, \infty)$



$f(x) = \sqrt{x}$

Natural Log function

D: $[0, \infty)$
R: $[3, \infty$



$f(x) = \ln x$

$$f(x) = \begin{cases} -2 & \text{if } -2 \le x < -1 \\ -1 & \text{if } -1 \le x < 0 \\ 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x < 2 \\ 2 & \end{cases}$$

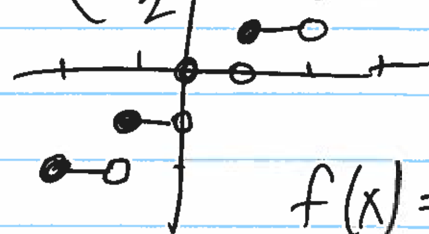
3 if $2 \le x \le 3$

Greatest Integer function (Step-function)

discontinuous

$D: [-2, 3)$

$R: -2, -1, 0, 1, 2$



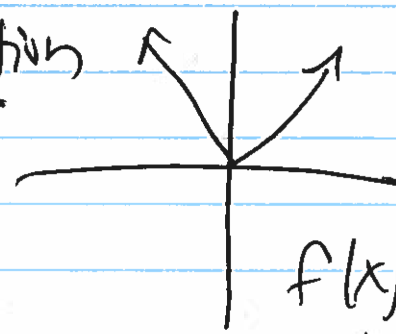
$f(x) = \text{int } x$

Absolute value function

Symmetric

$D: (-\infty, \infty)$

$R: [0, \infty)$

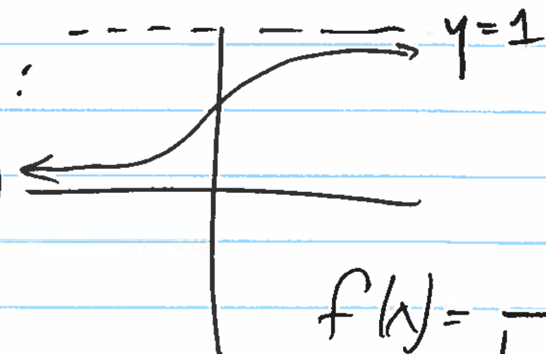


$f(x) = |x|$

absolute value sign

Logistic Function:

(biology & business)



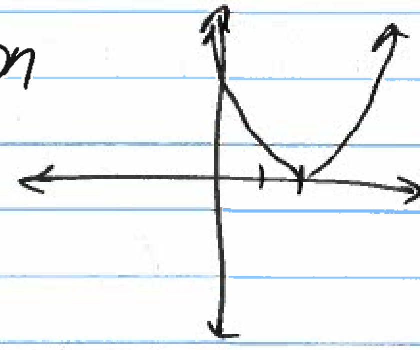
$D: (-\infty, \infty)$

$R: [-0.5, 1.5]$

$$f(x) = \frac{1}{1 + e^{-x}}$$

Analyzing Functions Graphically

ex 1 Graph the function
$y=(x-2)^2$



1.) What intervals

Increasing: $[2,\infty)$

Decreasing: $[-\infty, 2]$

2.) Function even, odd, or neither?

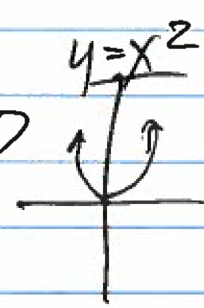
neither not symmetric with y-axis, x-axis, or origin.

3.) Function have any extrema?

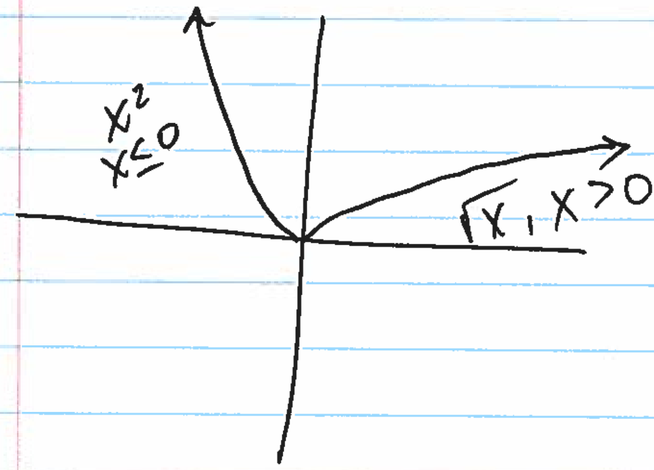
min: $(2,0)$

4.) How does graph relate to parent function $y=x^2$?

Moved 2 units right



ie2 $f(x) = \begin{cases} x^2, & \text{if } x \le 0 \\ \sqrt{x}, & \text{if } x > 0 \end{cases}$

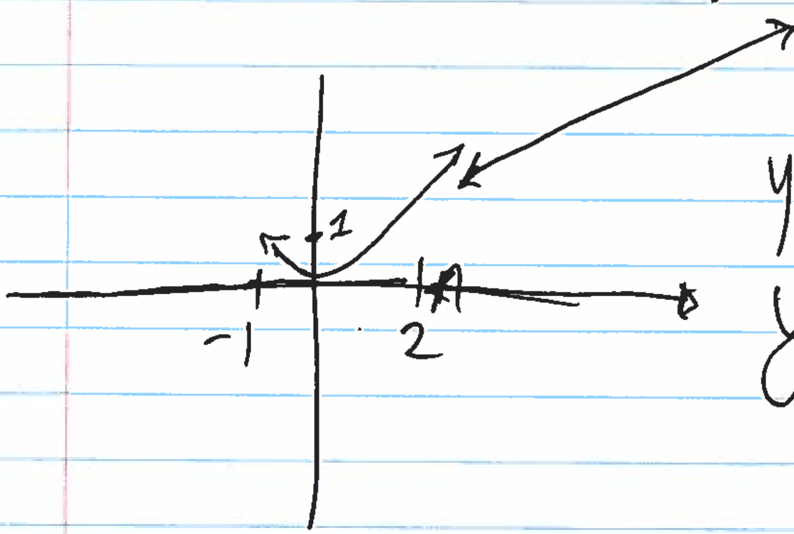


2ND math

$y = (x^2)(x \le 0)$

$y = (\sqrt{x})(x > 0$

ie3 $f(x) = \begin{cases} x^2, & -1 \le x < 2 \\ x+1, & x > 2 \end{cases}$



$y = (x^2)(-1 \le x)(x < 2)$

$y = (x+1)(x > 2)$

1.4 Building Functions from Functions

Rules of functions

Sum: $(f+g)(x) = f(x) + g(x)$

Difference: $(f-g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x) \cdot g(x)$

Quotient: $\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)}, \ g \neq 0$

Composition of functions:

(operation for combining functions using substitution)

$(f \circ g)(x) = f(g(x))$ ← substituted where x is located on $f(x)$

$(g \circ f)(x) = g(f(x))$ ← substituted where x is located on $g(x)$

ie1 $f(x) = e^x$ and $g(x) = \sqrt{x}$

Find $(f \circ g)x$ and $(g \circ f)(x)$

$(f \circ g)x$

$f(g(x))$

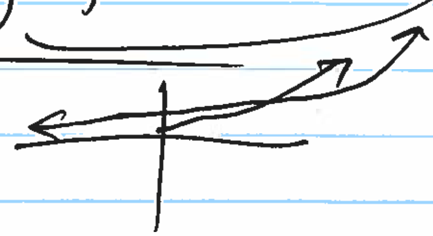
$f(x) = e^{(\sqrt{x})}$

$= e^{\sqrt{x}}$

$(g \circ f)(x)$

$g(f(x))$

$g(x) = \sqrt{e^x}$



Decomping Functions : (Reverse the process)

(ie1) $h(x) = (x+1)^2$ ~~and~~ so than $h(x) = f(g(x))$

$f(x) = x^2$

$g(x) = x+1$

(ie2) $h(x) = \sqrt{x^3+1}$ so that $h(x) = f(g(x))$

$f(x) = \sqrt{x}$

$g(x) = x^3+1$

a) Volume after t seconds is

$$V = 44t$$

b.) $\frac{4}{3}\pi r^3 = V$

volume of a sphere

$$\cancel{\frac{3}{4}}\left(\cancel{\frac{4}{3}}\right)\frac{\cancel{\pi}}{\pi} r^3 = V \cdot \frac{3}{4} \cdot \frac{1}{\pi}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

c) after 5 seconds $t = 5$ seconds

$$r = \sqrt[3]{\frac{3 \cdot (44t)}{4\pi}} = r = \sqrt[3]{\frac{3 \cdot 11t}{\pi}}$$

$$r = \sqrt[3]{\frac{3 \cdot 11(5)}{\pi}} = \sqrt[3]{\frac{3 \cdot 55}{\pi}} = 3.74 \text{ mm}$$

7

$(1,3)\quad x^2y + y^2 = 5$

$1^2(3) + (3)^2 = 5$

$3 + 9 = 5$

$12 \neq 5$ NO

$(2,1)\quad x^2y + y^2 = 5$

$2^2(1) + (1)^2 = 5$

$4 + 1 = 5$

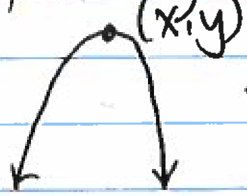
$5 = 5$ yes

Not a function; not all solutions work; plus x is used 2 times 2

1.5 Parametric Relations & Inverses

Parametric: introduce an extra independent variable usually t; which is time. which is a parameter.

ex) Throwing a ball in the air



t = time to hit the ground.

Defining a Function Parametrically

$x = t + 1$ $\quad$ t = any real #

$y = t^2 + 2t$

a) Find points of $t \Longrightarrow$ -3

-2

-1

0

1

2

3

B) Find algebraic relationship b/t x & y. (Eliminating the parameter)

$x = t+1$ start

$t = x-1$ finish

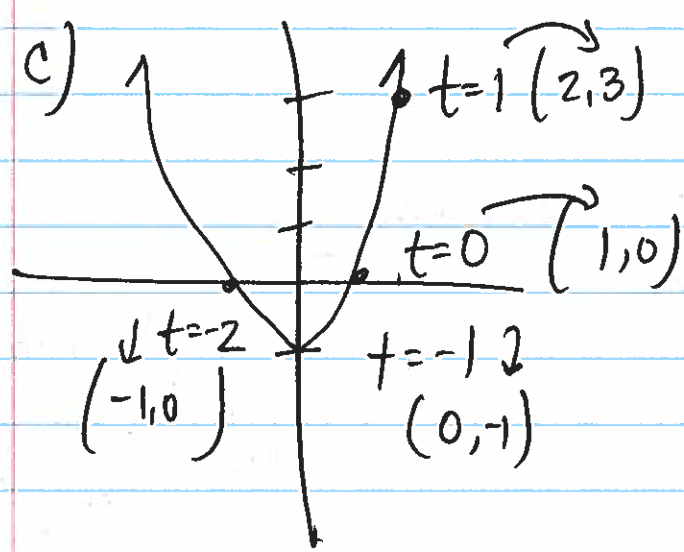
$y = t^2+2t$

$y = (x-1)^2 + 2(x-1)$

$y = x^2-2x+1+2x-2$

$\boxed{y = x^2-1}$

c)



graph

Cool stuff!

see your $\frac{\text{calc.}}{\text{graph}}$

T	X_{1T}	Y_{1T}
-3	-2	3
-2	-1	0
-1	0	-1
1	1	0
2	2	3
3	4	15

$\boxed{\text{ie 2}}$ $x = 4 - t \quad y = 2t$

Calculator Buttons

① mode → $\boxed{\text{PARAMETRIC}}$ → enter

② $\boxed{y=}$ → $X_{1T} = \underline{4 - t}$ type in

→ $Y_{1T} = \underline{2T}$ type in

③ Window → Set up as

−10	Tmin
10	Tmax
.1	Tstep
−10	X-min
10	Xmax
1	Xscl
−10	Ymin
10	Ymax

→ ~~[illegible]~~ graph

④ Trace (to get coordinates) or use table

⑤ 2ND → TBLSET Set up as:

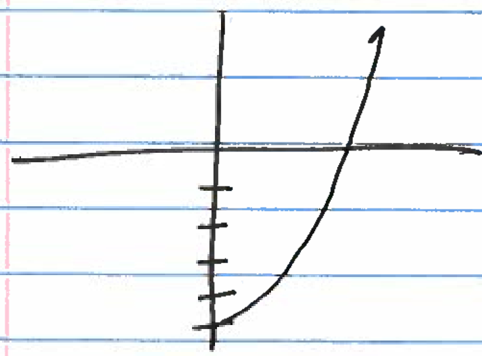
0
1
Auto
Auto

Think of T as time

ie3 $x=\sqrt{t}$; $y=t-5$, $t \geq 0$

only change ⇒ Windows

Tmin = 0



T	X_1T	Y_1T
0	0	−5
1	1	−4
2	1.41	−3
3	1.73	−2
4	2	−1

* Watch for those that specify a T

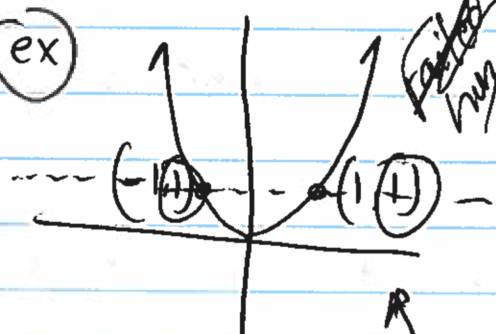
Day 2

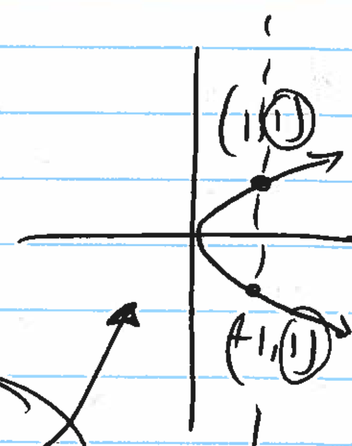
Inverse Relations & Inverse Functions

→ the ordered pair (a,b) is in a relation iff the ordered pair (b,a) is in the inverse relation.

→ The inverse of a relation is a function iff each horizontal line intersects the graph of the original relation in at most 1 point

ex





failed test

Cannot double y-values on inverse

all same point (not inverse)

original

① $y = \dfrac{x}{x+1}$ $\qquad y' \Rightarrow x = \dfrac{y}{y+1}$

② ~~substitute x-value into original~~ mult. by dinominato

$(y+1)\,\cancel{x} = \left(\dfrac{y}{\cancel{y+1}}\right)\cancel{(y+1)}$

$$f'(x) = \frac{x}{1-x}$$

③ $x(y+1) = y$

$xy + x = y$
$-y \quad -x \quad -y \quad -x$

④ $xy - y = -x$

⑤ GCF $\dfrac{y(x-1)}{x-1} = \dfrac{-x}{x-1}$

⑥ Divide

$y = \dfrac{-x}{x-1}$ ⑬

mult by (-1)

$y = \dfrac{(-x)}{(x-1)}(-1)$

$y = \dfrac{x}{-x+1}$

$y' = \dfrac{x}{1-x}$ inverse

Verifying Inverse Functions
(must equal same)

from ie2

$f(x)$

$y = \sqrt{x+3}$ $\qquad$ $g(x) = x^2 - 3$

$f(g(x))$ $\qquad$ $g(f(x))$

$f(g(x)) = \sqrt{x^2 - 3 + 3}$ $\qquad$ $g(f(x)) = \sqrt{x^2 \cancel{+3-+3}}$

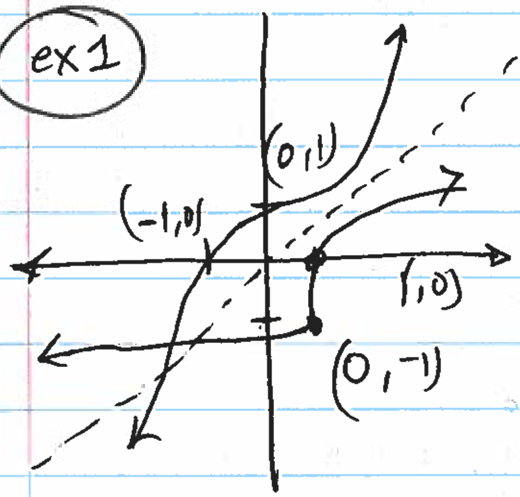
$= \sqrt{x^2}$ $\qquad$ $= \sqrt{x^2}$

$= x$ $\qquad$ $= x$

inversed & proved

Finding Inverse Function Graphically

ex 1



the mirror $y = x$ (Reflect about that point)

$$f(x) = (x, y)$$

$$f'(x) = (y, x)$$

$$f(x) = (-1, 0) \ \& \ (0, 1)$$

$$f'(x) = (0, -1) \ \& \ (1, 0)$$

1.6 Graphical Transformations

Transformations: movement of a graph.

Rigid: leave shape & size

Non-rigid: do not leave shape and/or size

Vertical Translations: shift graph up or down

$y = f(x) \pm c$

Horizontal Translations: shift graph left or right

$y = f(x \pm c)$

ie1 $y = |x| - 4$

down 4 units

ie2 $y = |x+2|$

left 2 units

$$* \quad \begin{aligned} x+2 &= 0 \\ -2 \; &\; -2 \\ \hline x &= -2 \end{aligned}$$

Reflections Across Axes (coordinates)

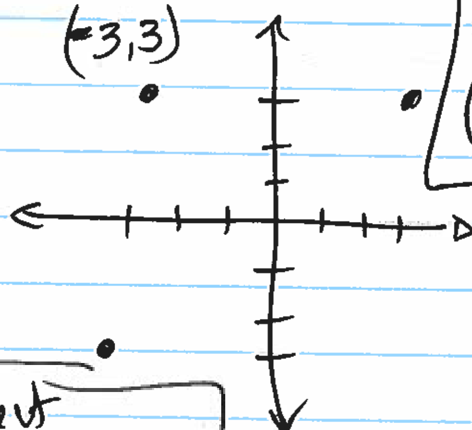
original (x, y)

Reflect x-axis $(+x, -y)$

Reflect y-axis $(-x, y)$

ex

original $(-3,3)$



Reflection y-axis $(3,3)$

Reflect x-axis $(-3,-3)$

Reflections For Functions

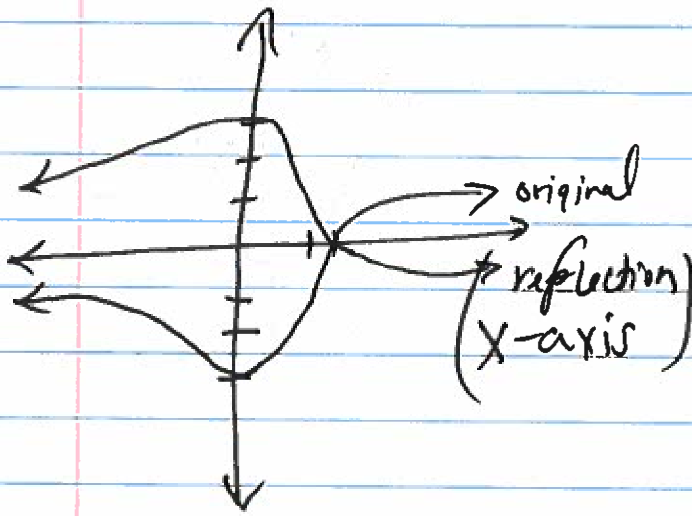
Across x-axis

$y = -f(x)$ (opposite of equation)

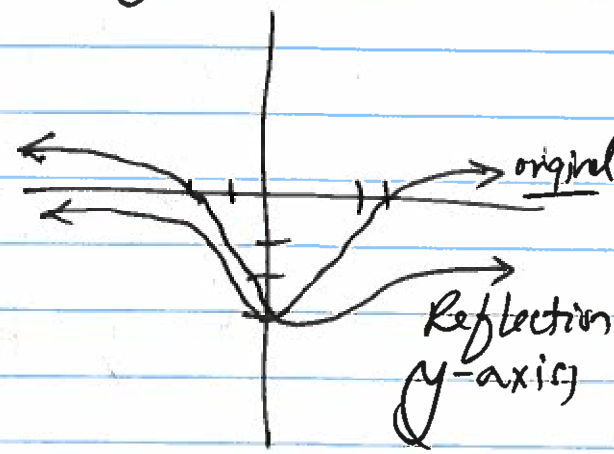
Across y-axis

$y = f(-x)$ (opposite of x values)

X-axis $-f(x)$
graph



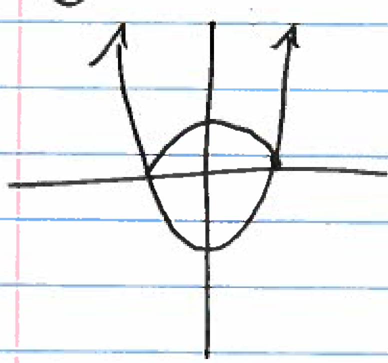
y-axis $f(-x)$



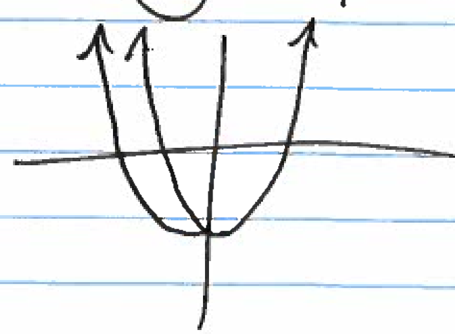
skip 2020-2021

Graphing Absolute Value Compositions

ex1 $y = |f(x)|$



ex2 $y = f(|x|)$



Vertical & Horizontal Stretches & Shrinks

* $\boxed{C \rightarrow \text{positive } \#}$

FORMULA

Horizontal Stretches or Shrinks

$$y = f\left(\frac{x}{c}\right)$$

stretch if $c > 1$
shrinks if $c < 1$

Vertical Stretch or Shrink

$$y = c \cdot f(x)$$

stretch $c > 1$
shrink $c < 1$

ie1 Let C_1 be the curve of

$$y_1 = f(x) = x^3 - 16x$$

a) Vert. Stretch of C_1 by factor of 3

b) hor. shrink of C_1 by a factor of $1/2$

Combining Transformations in Order

ie 1 Curve original $y=x^2$

a) Horizontal Shift 2 units right
b) vertical stretch factor of 3
c) vertical translation 5 units up

a) $y=x^2$

$y=(x-2)^2$

2 units right

b/c $x-2=0$
$+2 \quad +2$
$x=2$

b) $y=c(f(x))$ ← vert. stretch form

$y=3(x-2)^2$

c) up 5

$\boxed{y=3(x-2)^2+5}$

I am good with this!

expanded solution if asked...

$3(x-2)(x-2)+5$

$3(x^2-4x+4)+5$

$3x^2-12x+12+5$

$=3x^2-12+27$

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