4.1 Angles and Their Measures

(D) Degree: angle measure by symbol (°)
$\frac{1}{180}$TH of a straight angle

(m) Minute: (denoted by ') each degree is divided in 60 minutes

(S) Seconds: (denoted by ") 60 seconds in a minute

Ex 1 Worthing with DMS

a) Convert 37.425° to DMS

$$0.425\left(\frac{60'}{1}\right) = 25.5'$$

$$0.5'\left(\frac{60''}{1}\right) = 30''$$

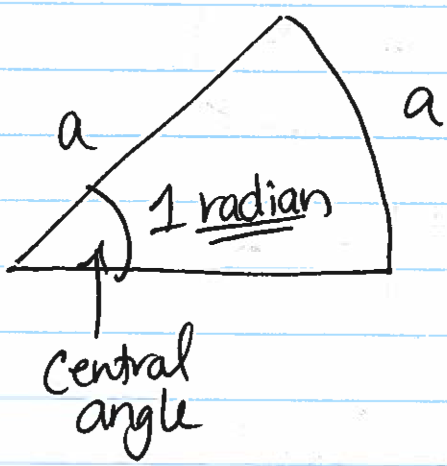
$$37° 25' 30''$$

b) $42°34'36''$ to degrees

$$42° + \left(\frac{24}{60}\right)° + \left(\frac{36}{3600}\right) = \boxed{42.41°}$$

<u>Course or bearing</u>: angle of the line of travel measured clockwise from due north.

<u>Radian</u>: Central angle of a circle if it ~~it~~ intercepts an arc with the same length as the radius.



P2 Working with radian Measure

a) ~~How many radians are in 90°~~

$$\boxed{\text{FORMULA: degree} \left(\frac{\pi}{180}\right)}$$ degree to radians

$$\boxed{\text{FORMULA: Radian} \left(\frac{180}{\pi}\right)}$$ radians to degrees

a) How many Radians in 90°

$$90 \cdot \left(\frac{\pi}{180}\right) = \frac{90\pi}{180} = \frac{\pi}{2} \text{ radian}$$

b) $\frac{\pi}{3}$ radians to degrees

$$\frac{\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}} = \frac{180}{3} = 60°$$

Circular Arc Length

Arc Length Formula (When you have Radian Measure)

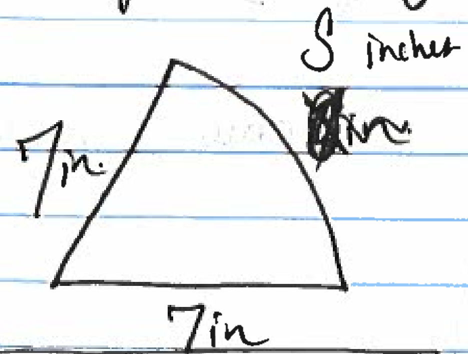
$$S = r\theta$$

θ = angle

Arc Length Formula (When you have degree measure)

$$S = \frac{\pi r \theta}{180^\circ}$$

P3 Perimeter of a Slice of Pizza



Find the perimeter of a 60° slice of large 7 in radius pizza.

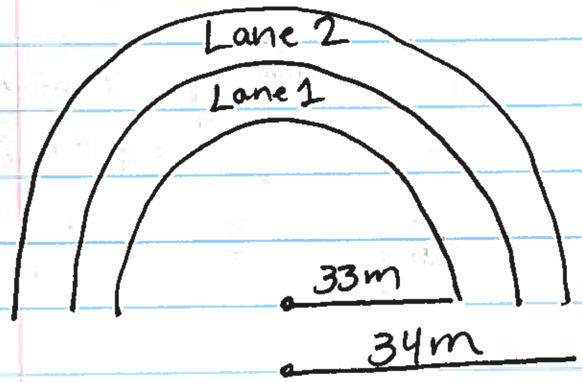
Formula: $S = \frac{\pi r \theta}{180°}$

$$S = \frac{\pi(7)(60°)}{180}$$

$$S = \frac{\pi(7)}{3} = \frac{7\pi}{3} \approx 21.3\text{ in}$$

Day 2

P4 Designing a Running Track



Lanes 1 meter wide

Each lane semicircle with central ∠ = π

$$S = r\theta$$

$$S = r\pi$$

$$34\pi - 33\pi = \pi \approx 3.14 \text{ meters longer}$$

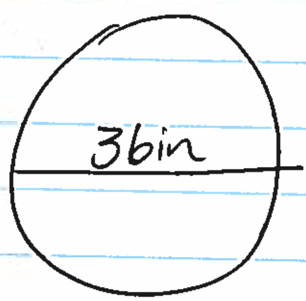
not sure to keep?

Angular & Linear Motion

angular speed: measured in units like revolutions per minute

Linear speed: measured in units like miles per hour

P5 Using Angular Speed



630 rpm (revolutions per minute)

Find trucks speed in mph

① convert revolution per minute to mph

$$\frac{630\text{ rev}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{2\pi\text{ radians}}{1\text{ rev}} \times \frac{18\text{ in}}{1\text{ radian}} \times \frac{1\text{ ft}}{12\text{ in}} \times \frac{1}{5280\text{ ft}} = 67.47\text{ mi/hr}$$

7

* A nautical mile is 1' of the Earth circumference at the equator

Distance Conversions

1 Statute mile $\hat{=}$ 0.87 nautical mile

1 nautical mile $\hat{=}$ 1.15 Statute mile

Water mile — land miles

Ex 6 Converting to nautical miles

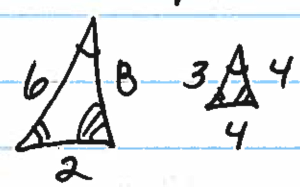
2698 statute miles to nautical miles

$$\begin{array}{r} 2698 \\ \times\ .87 \\ \hline 2347.26 \end{array}$$

$\hat{=}$ 2347 nautical miles

4.2 Trig. Functions of Acute Angles

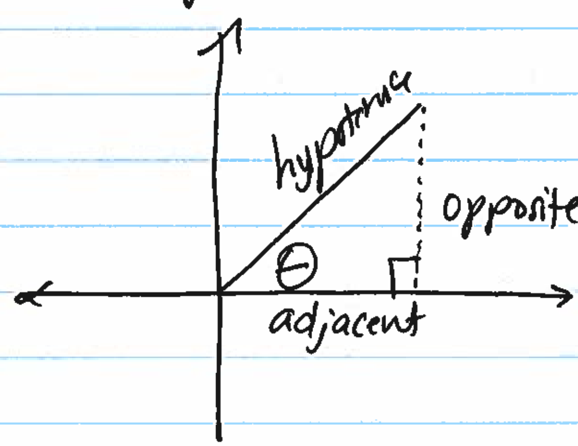
Similar: shapes with same shape & proportional sides

ex 

Corr. ∠'s
sides proportion
all $= \frac{2}{1}$

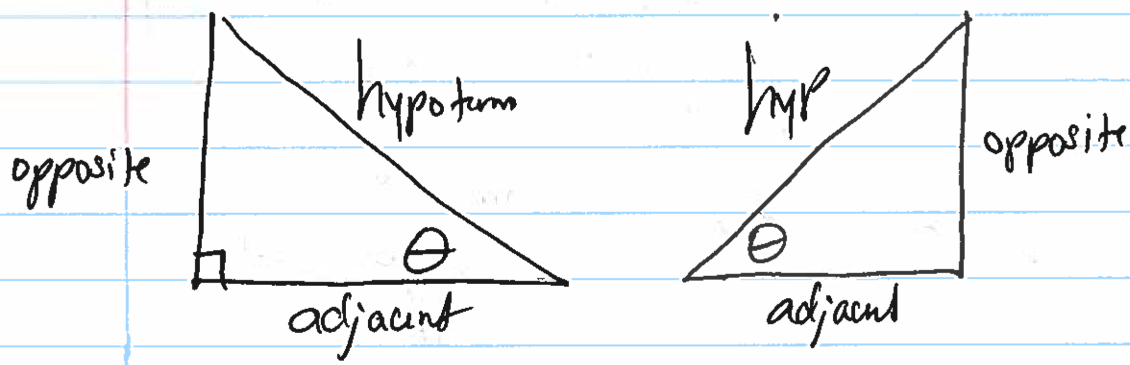
Right Δ trig: Creates right triangles & utilize sine, cosine, tangent cosecant, secant, cotangent to solve.

Standard position: (acute angle in this example) at Θ



w/ Right angle created with vertical dashed line

Triangle Referenced Sides WRT to θ



Trig. Functions

(SOH) Sine θ (sin θ) $= \dfrac{\text{opposite}}{\text{hypotenuse}}$

(CAH) cosine θ (cos θ) $= \dfrac{\text{adjacent}}{\text{hyp.}}$

(TOA) Tangent (tan θ) $= \dfrac{\text{opposite}}{\text{adjacent}}$

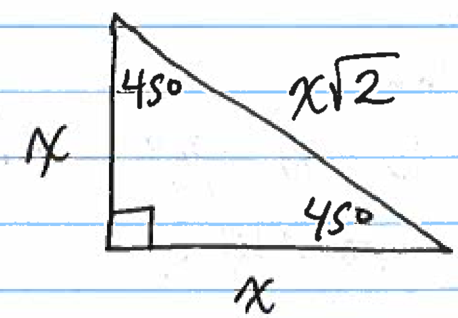
(sines ~~i~~ reciprocal) Cosecent θ = csc $\theta = \dfrac{\text{hyp}}{\text{opp}}$

(cosine ~~~~ reciprocal) Secant θ = sec $\theta = \dfrac{\text{hyp}}{\text{adj}}$

(tangent reciprocal) cotantangent θ = tan $\theta = \dfrac{\text{adj}}{\text{opp}}$

Ex P1 Evaluating Trig. function of 45°

Remember
45°-45°-90°
Δ FORMULA $\Rightarrow$



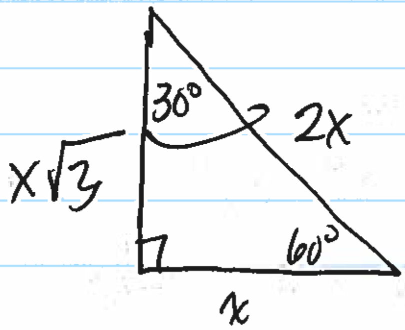
$\sin 45^\circ = \dfrac{x}{x\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ (SOH) $\qquad \csc 45^\circ = \dfrac{\sqrt{2}}{1}$

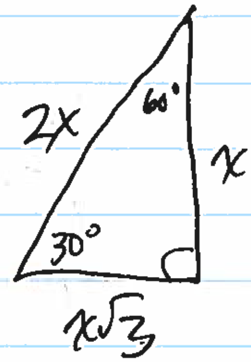
$\cos 45^\circ = \dfrac{x}{x\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ (CAH) $\qquad \sec 45^\circ = \dfrac{\sqrt{2}}{1}$

$\tan 45^\circ = \dfrac{x}{x} = 1$ (TOA) $\qquad \text{cotan } 45^\circ = 1$

P2 Trig. Function of 30°

Reminder
30°-60°-90°
FORMULA





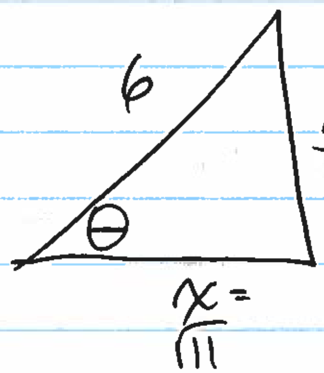
$\sin 30^\circ = \frac{x}{2x} = \frac{1}{2}$ (SOH) $\qquad \csc 30^\circ = \frac{2}{1}$

$\cos 30^\circ = \frac{\cancel{x}\sqrt{3}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$ (CAH) $\qquad \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{\cancel{x}}{\cancel{x}\sqrt{3}} \quad \frac{1}{\sqrt{3}}$ (TOA) $\qquad \cot 30^\circ = \frac{\sqrt{3}}{1}$

B3 Use One Trig Ratio to Find them ALL

ie1



① Find missing part

$a^2+b^2=c^2$

$5^2+x^2=6^2$

$25+x^2=36$

$-25 \quad\quad -25$

$\sqrt{x^2}=\sqrt{11}$

$\sin\theta = \frac{5}{6}$ (SOH)

$\csc\theta = \frac{6}{5}$

$\cos\theta = \frac{\sqrt{11}}{6}$ (CAH)

$\sec\theta = \frac{6}{\sqrt{11}}$

$\tan\theta = \frac{5}{\sqrt{11}}$ TOA

$\cot\theta = \frac{\sqrt{11}}{5}$

Trig. Functions on a Calculator

① using calc. in the wrong mode (degree/radians)

ie1 $\sin(10^\circ) = -0.544$ ← in-correct

must be in degrees ~~to get a~~ $\sin(10^\circ) = 0.1736$

Inverse Trig Keys to evaluate cot, sec, csc

ie2 $\cot 30^\circ = 1.732$

type in ⇒ $\frac{1}{\tan 30^\circ}$

↑ in degree mode

* Tan^{-1} – is the inverse when you switch x & y values. We will look into this later

Closing Parathesis required...

ie 1 $\sin(30)+2$

incorrect ⇒ $\sin(30+2) = 0.5299$

Correct ⇒ $\sin(30)+2 = 2.5$

P4 Getting the exact Answer on a calculator...

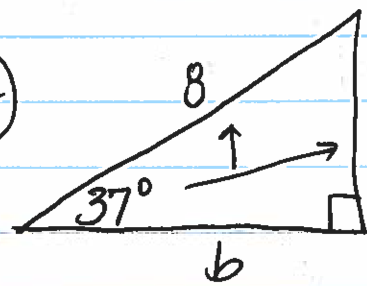
ie 1 $\cos 30° = 0.866025403B$

degree mode

0.8660

Solving a Right Δ (missing sides)

ie1



$a = 4.815$

a) $\underline{\sin 37°} = \frac{O}{H} \quad \frac{a}{8}$

$8\,(0.6018) = \left(\frac{a}{8}\right) 8$

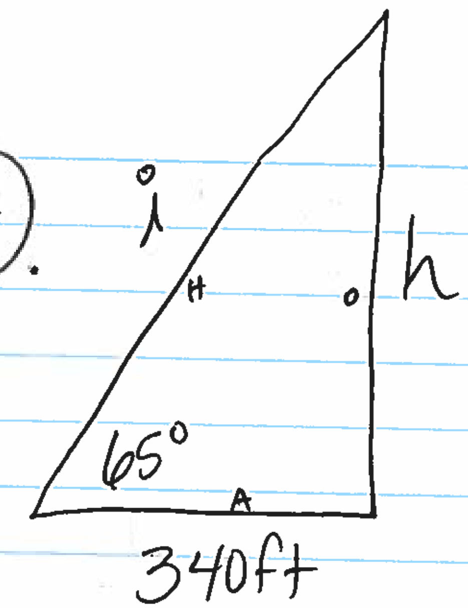
$a = 4.815$

b) $\underline{\cos} \; 37° = \frac{A}{H} \quad \frac{b}{8}$

$8\,(0.7986) = \left(\frac{b}{\cancel{8}}\right) 8$

$b \approx 6.3891$

ie 2



h) tan ~~SA~~ $65° = \frac{O}{A} \quad \frac{h}{340}$

$340(2.1445) = \left(\frac{h}{340}\right) 340$

$= 729.1324$

i) cos $65° \quad \frac{A}{H} \quad \frac{340}{i}$

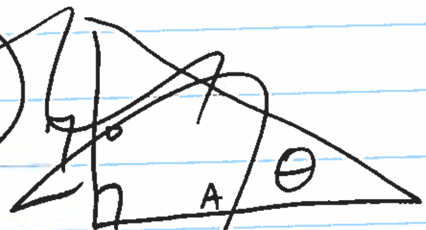
$i(0.4226) = \left(\frac{340}{i}\right) i$

$\frac{0.4226i}{0.4226} = \frac{340}{0.4226}$

$i = 804.5433$

~~Extra Practice~~ ~~(Find the angle with given sides)~~

ie1



ie2

$\cot\theta = \frac{1}{\sqrt{3}}$

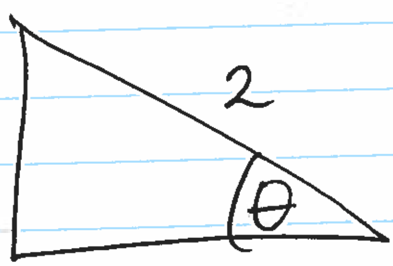
$\tan\theta = \sqrt{3}$

$\tan^{-1}(\sqrt{3}) = 60°$

$\cancel{\tan\theta = \frac{7}{11}}$

$\cancel{\tan\theta = 0.63636}$ $= 32°$

ie1



$\sin\theta = \frac{1}{2}$

$\sin\theta = 0.5$

$\theta = 30°$

$\boxed{\text{degree}}$

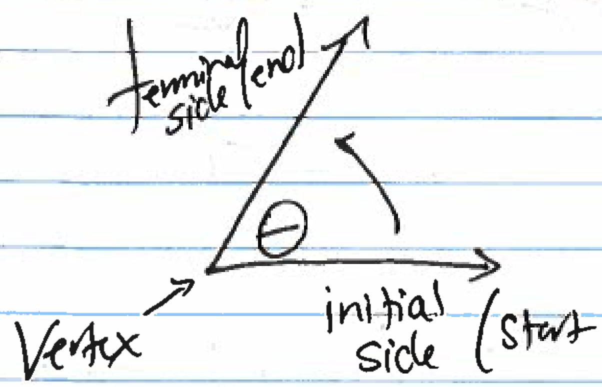
2nd

sin

$\cancel{(\sin 0.5)^{-1} = 30°}$

$\sin^{-1}(0.5) = 30°$

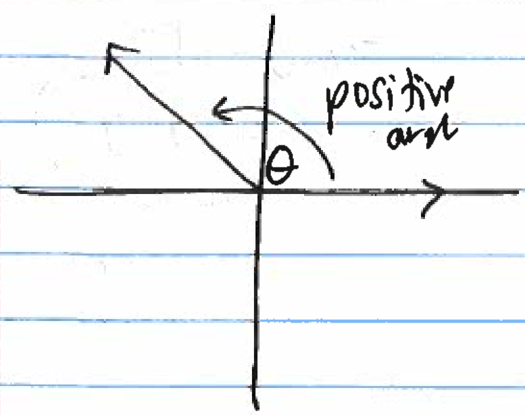
4.3 Trig Extended: Circular Functions

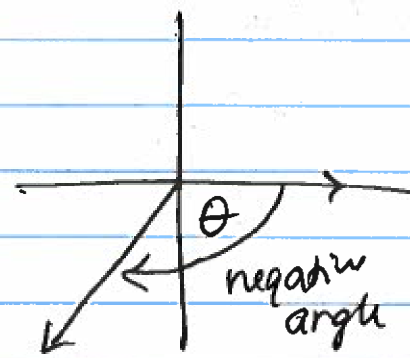


positive angles:
counterclockwise rotations

negative angle
clockwise rotation

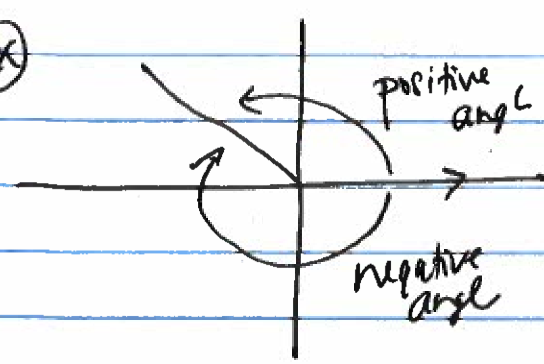
Standard Position





Coterminal angles: same initial & terminal sides only from counterclockwise & clockwise movement

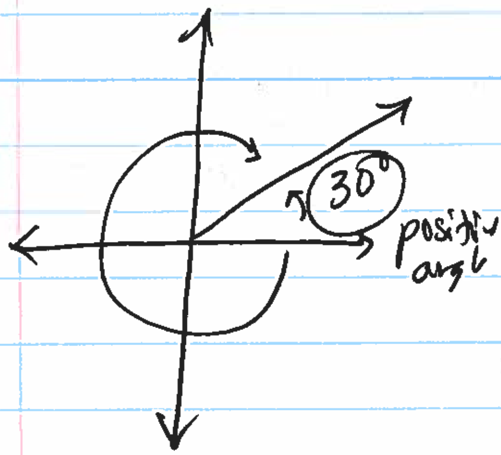
(ex)



(1)

P1 Finding Coterminal Angles

a) 30°

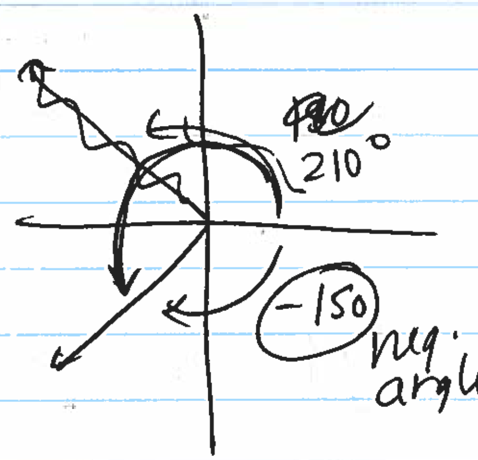


$30 - 360$

$= -330$

negative angle

b) -150



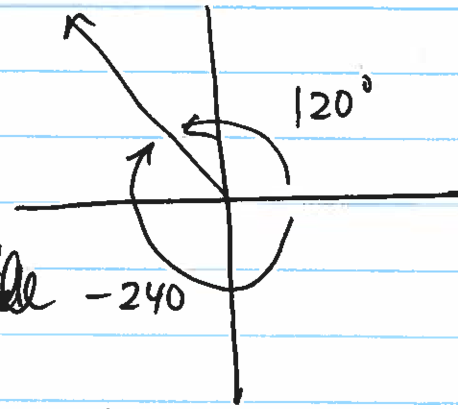
$360 - 150$

$= 210^\circ$

pos. angle

c.) $\frac{2\pi}{3}$ Radians

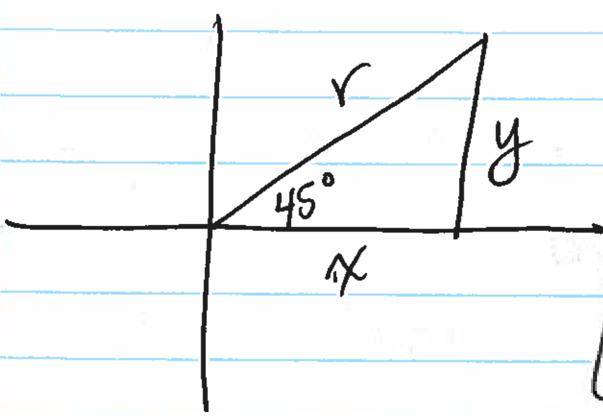
or $\frac{2(180)}{3} = 120°$ positive angle



$120 - 360 = -240$ ← negative angle

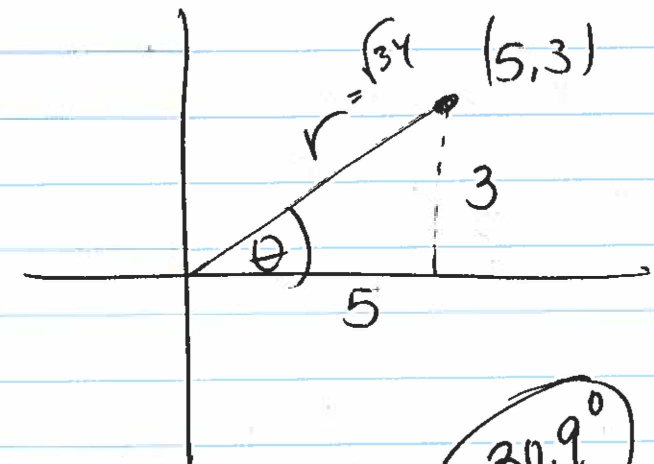
or $\frac{-4\pi}{3}$

$\boxed{P2}$ Evaluating Trig. Functions Determined by a Point in Quadrant 1.



Remember:
$a^2+b^2=c^2$
or $x^2+y^2=r^2$

ic1 Let θ be the acute angle whoa terminal side contains (5,3) Find 6 trig functions of θ



pythagorean theorem

$x^2+y^2=r^2$

$3^2+5^2=r^2$

$9+25=r^2$

$\sqrt{34} = \sqrt{r^2}$

$r=\sqrt{34}$

$=30.9^\circ$

$\sin\theta = \frac{3}{\sqrt{34}} = 0.514$ $\csc\theta = \frac{\sqrt{34}}{3} = 1.944$

31°

$\cos\theta = \frac{5}{\sqrt{34}} = 0.857$ $\sec\theta = \frac{\sqrt{34}}{5} = 1.166$

30.9°

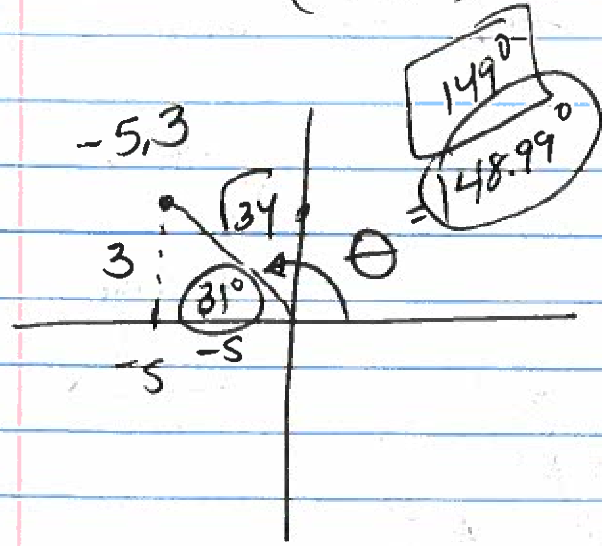
$\tan\theta = \frac{3}{5} = 0.6$ $\cot\theta = \frac{5}{3} = 1.667$

2nd sin cos tan decimal value

angle

~~angle~~

ie2 Let θ any ~~acute~~ whose terminal side contains $(-5,3)$ Find 6 trig. functions of θ



$$x^2+y^2=r^2$$

$$3^2+(-5)^2=r^2$$

$$\sqrt{34}=\sqrt{r^2}$$

$$r=\sqrt{34}$$

30.9°

$\sin\theta = \frac{3}{\sqrt{34}} = 0.514 \quad \csc\theta = \frac{\sqrt{34}}{3} = 1.944$

148.93

$\cos\theta = \frac{-5}{34} = -0.857 \quad \sec\theta = \frac{\sqrt{34}}{-5} = -1.166$

opposite direction

$\tan\theta = \frac{3}{-5} = -0.6 \quad \tan\theta = \frac{-5}{3} \approx -1.667$

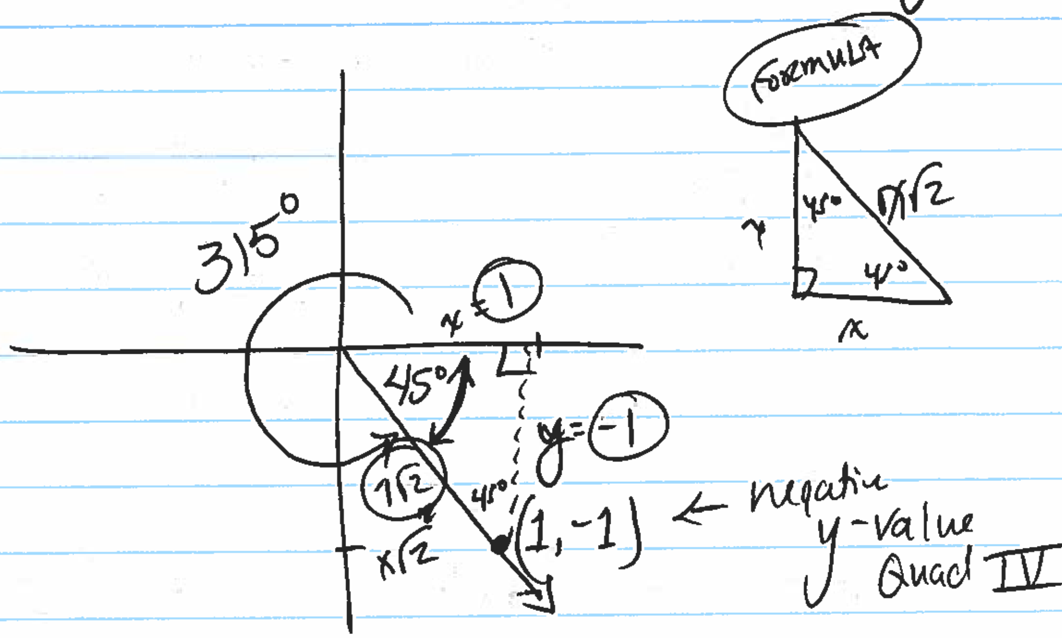
-30.96

~~$\sin\theta = \frac{y}{r} \quad \tan\theta = \frac{y}{x} \quad \sec\theta = \frac{r}{x}$~~

~~$\cos\theta = \frac{x}{r} \quad \csc\theta = \frac{r}{y} \quad \cot\theta = \frac{x}{y}$~~

$\boxed{P4}$ Evaluating the Trig Function of 315°

Find the 6 trig. functions of 315°



$\sin 315 = \dfrac{-1}{\sqrt{2}}$ $\qquad$ $\csc \theta = \dfrac{\sqrt{2}}{-1}$

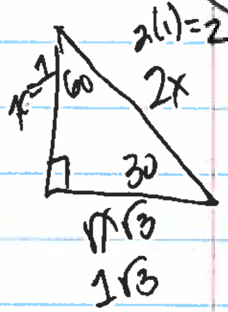
$\cos 315° = \dfrac{1}{\sqrt{2}}$ $\qquad$ $\sec \theta = \dfrac{\sqrt{2}}{1}$

$\tan 315° = \dfrac{-1}{1}$ $\qquad$ $\text{cotan} = \dfrac{1}{-1}$

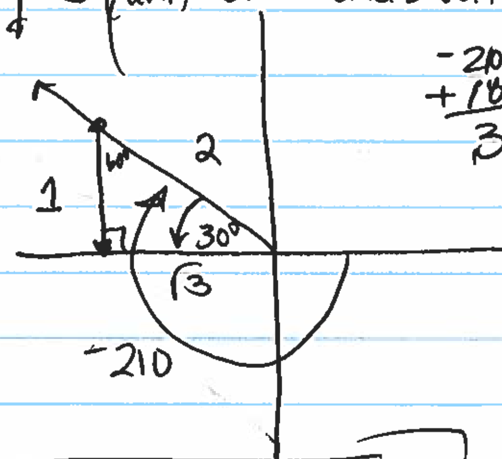
P5 Evaluating More Trig. Functions

ie1 $\sin(-210°)$

* $y=1$ (unit circle ends with $y=1$ or $y=-1$)



$30°-60°-90°$
refrence triange



$$\begin{array}{r} -210 \\ +180 \\ \hline 30° \end{array}$$

$x^2+y^2=c^2$

$x^2+1^2=2^2$

$x^2=4-1$

$\sqrt{x^2}=\sqrt{3}$

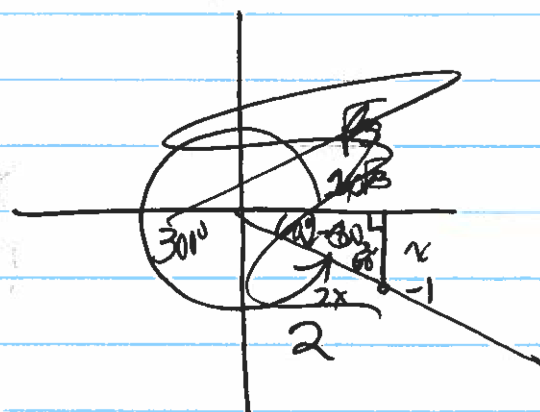
$\boxed{x=\sqrt{3}}$

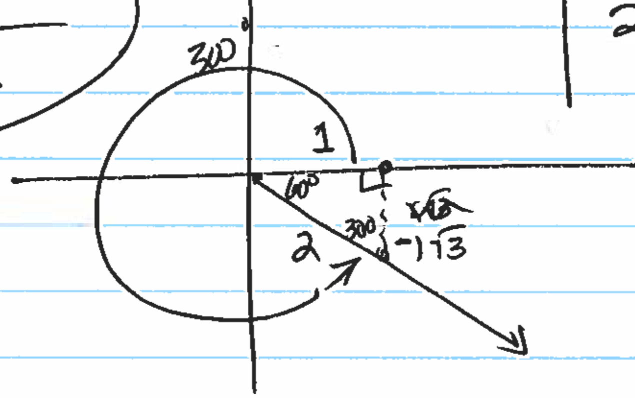
$$\boxed{\sin\theta = \frac{O}{H} = \frac{1}{2}}$$

ie2 $\tan\frac{5\pi}{3} = \frac{5(180)}{3}$

$5\cdot 60 = 300°$

$$\tan = \frac{O}{A} = \frac{-\sqrt{3}}{1}$$



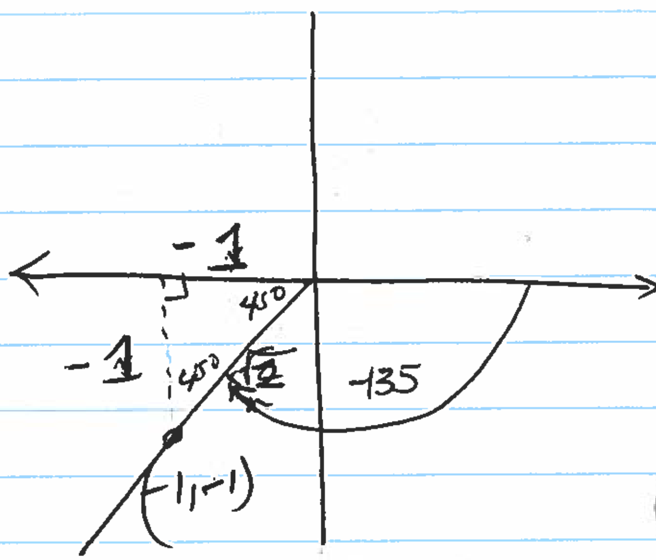


$\boxed{\text{ie3}}$ $\sec\left(-\frac{3\pi}{4}\right) = \frac{-3(180)}{4} = \frac{-3(\cancel{90}^{45})}{\cancel{2}}$

$\cos = \frac{A}{H}$

-135

$\sec = \frac{H}{A} = \frac{\sqrt{2}}{-1}$



$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{\cancel{x}}{\cancel{r}}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot an = \frac{x}{y}$

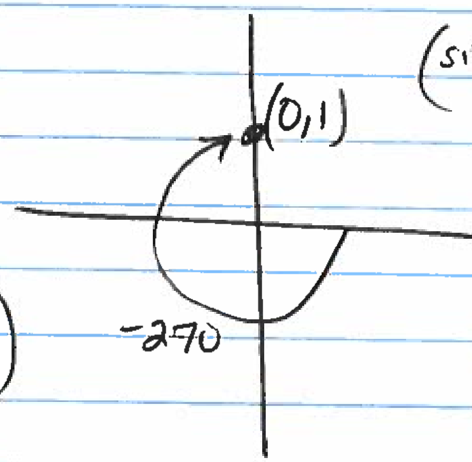
Quadrantal Angles: angles Whose terminal sides lie along one of the coordinate axes.

$\boxed{P6}$ Evaluating Trig. Functions of Quad. Angles

ie1 $\sin(-270^\circ) =$

$\sin -270 = \frac{0}{H} = \frac{y}{r}$

$\sin -270 = \frac{1}{1} = 1$



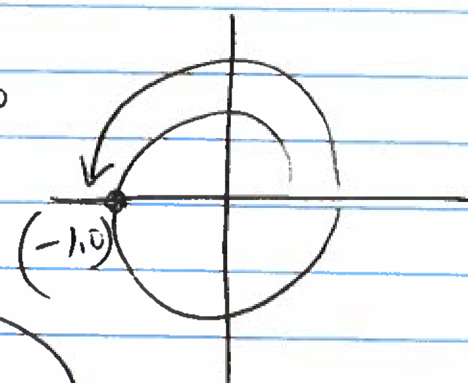
$(\sin)(-270) = 1$

ie2 $\tan 3\pi =$

$3 \cdot 180 = 540^\circ$

$\tan 3\pi = \frac{0}{A} = \frac{y}{x}$

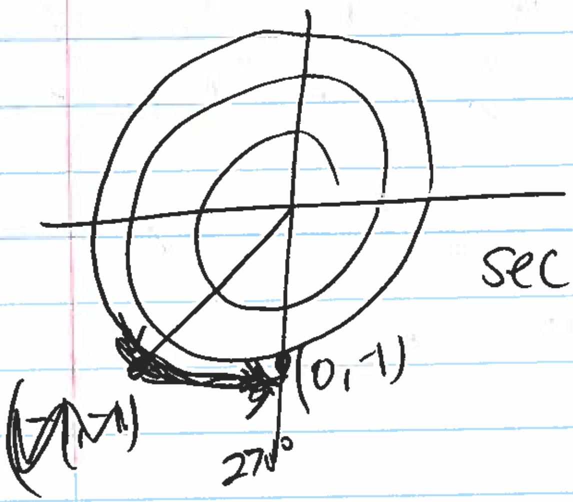
$\tan 3\pi = \frac{0}{-1} = 0$



ie3 $\sec \frac{11\pi}{2} = \frac{11(180)}{2} = 11 \cdot 90$

$$\begin{array}{r} 990^\circ \\ -720 \\ \hline 270^\circ \end{array}$$

$\cancel{360}$ left



$\sec = \frac{r}{x} = \frac{1}{0}$ = undefined

Find $\cos\theta$ & $\tan\theta$

P7 Using 1 Trig Ratio to Find the Others

→ located in either Q I or ~~Q II~~

ie1 $\sin\theta = \frac{3}{7}$ and $\tan\theta < 0$

$\tan\theta \frac{y}{x} = \frac{3}{-}$ (A)

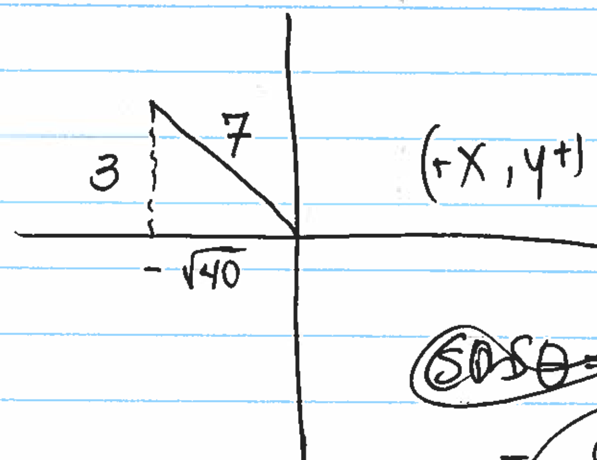
$x^2+y^2=r^2$

$x^2+3^2=7^2$

$x^2+9=49$

$\sqrt{x^2}=\sqrt{40}$

$x=\sqrt{40}$



$(+x, y+)$

~~$\cos\theta = \frac{\sqrt{40}}{7}$~~

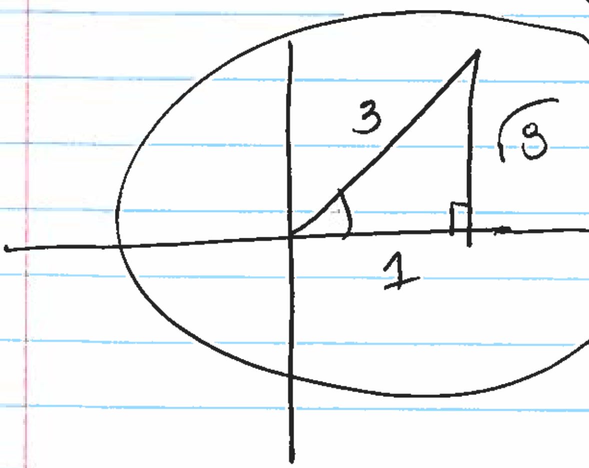
$\cos\theta = \frac{A}{H} = \frac{-\sqrt{40}}{7}$

$\tan\theta = \frac{O}{A} = \frac{3}{-\sqrt{40}}$

ie2 $\sec\theta = \frac{3}{1}$ & $\sin\theta > 0$

$\sec\theta = \frac{H}{A}$

$\sin\theta = \frac{y}{r} = \frac{+}{+3}$



$1^2+y^2=3^2$

$\sqrt{y^2}=\sqrt{8}$

$y=\sqrt{8}$

$\cos\theta = \frac{A}{H} = \frac{1}{3}$

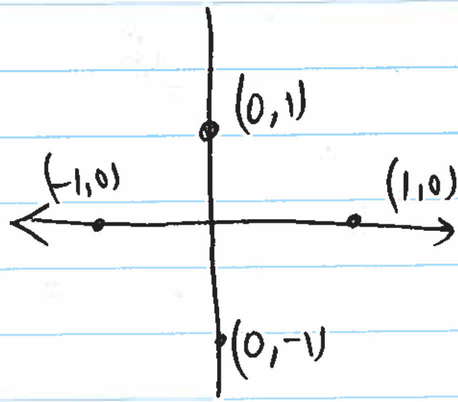
$\tan\theta = \frac{O}{A} = \frac{\sqrt{8}}{1}$

$\sec\theta = \frac{H}{A}$

$\cos\theta = \frac{A}{H}$

c) $\cot\theta$ is undefined a $\sec\theta$ is negative

$\cot\theta = \frac{x}{y}$ $\frac{1}{0}$ $\cdot$ $\frac{-1}{0}$ $\sec\theta = \frac{r}{x} = 0$



$\frac{A}{0}$

$$\boxed{\begin{aligned} \cos\theta &= -1 \\ \tan\theta &= \frac{0}{-1} \end{aligned}}$$

$\tan\theta = \frac{0}{A} = \frac{0}{-1}$

Day 2 (Very important)

Trig Functions of Real Number

$\sin\theta = y$ $\cos\theta = x$

$\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{1}{y}$ $\sec\theta = \frac{1}{x}$

$\cot\theta = \frac{x}{y}$

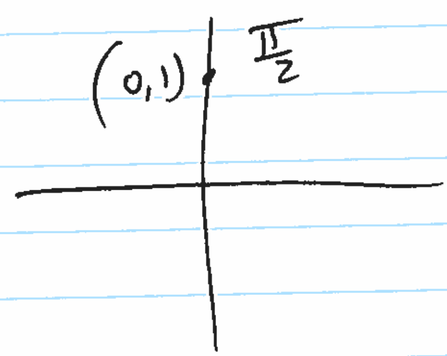
Periodic Functions:

$y = f(t)$ is periodic is a positive number c such that $f(t+c) = f(t)$. The smallest such number c is called <u>period</u> of the function.

PB Using Periodicity

Reminder
$\sin\theta = y_{\text{coordinate}}$

ie1 $\sin\left(\dfrac{57{,}801\pi}{2}\right)$



$\sin \dfrac{\pi}{2} + \dfrac{57801\pi}{2}$

$\sin \boxed{\dfrac{\pi}{2}} + \underbrace{28{,}900\pi}_{144} = \sin\dfrac{\pi}{2} = 1$

$y = 1$

$\underline{\underline{\cos \theta = x}}$ (coordinate)

b.) $\cos(288.45\pi) - \cos(280.45\pi)$

$\cos(280.45\pi + 8\pi) - \cos 280.45\pi = ☺$

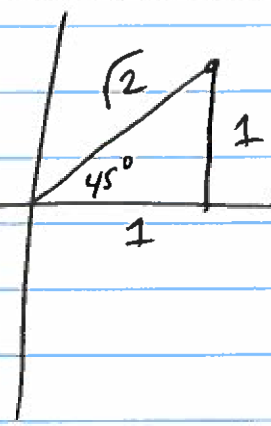
$\cos(280.45\pi)(8\pi - 1)$

$\boxed{\text{Cosine same on both}}$

c) $\tan\left(\frac{\pi}{4} - 99999\pi\right)$

$\boxed{\tan \theta = \frac{y}{x}}$

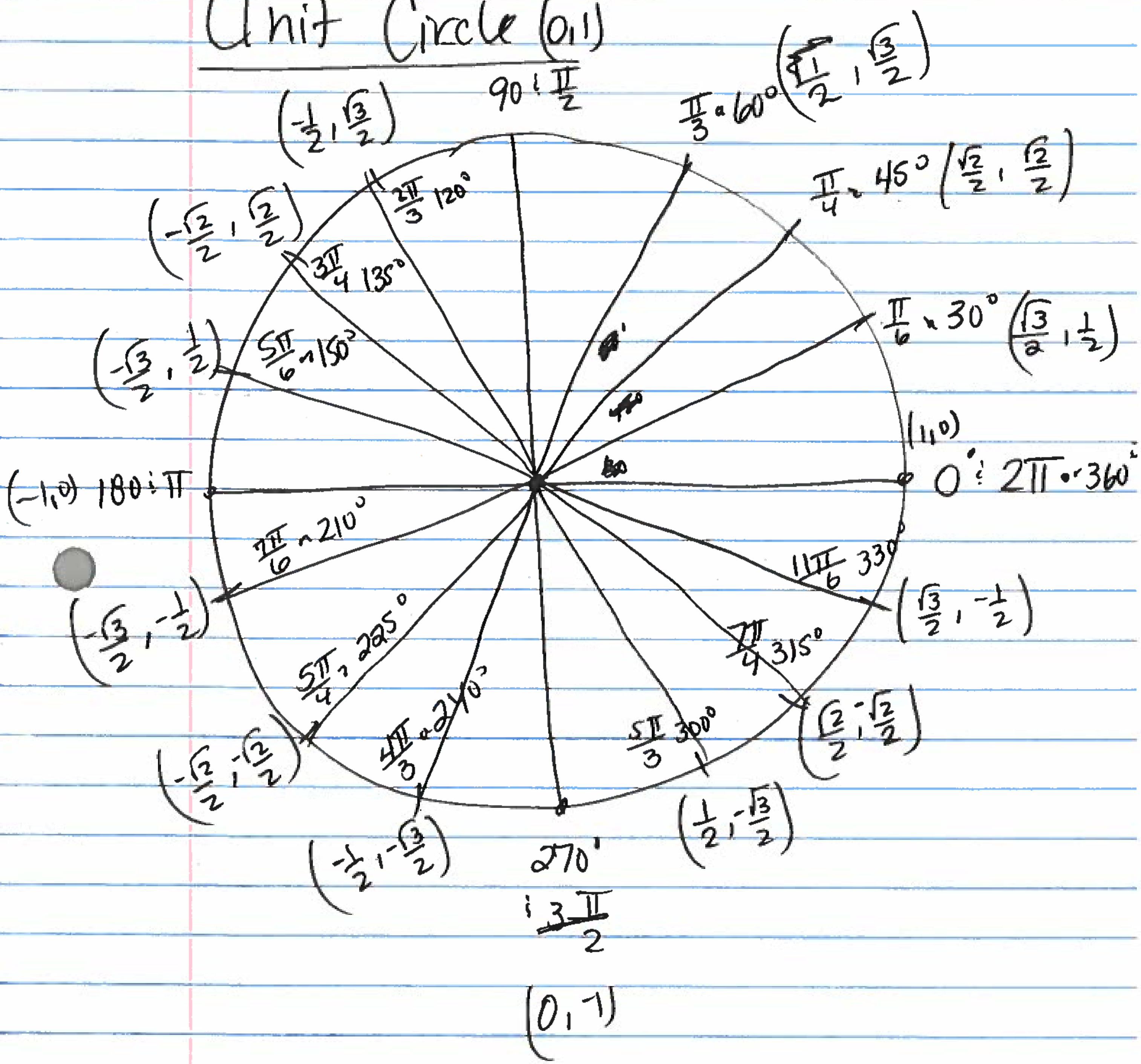
$= \tan \frac{\pi}{4}$ (45° same thm



$\tan = \frac{O}{A} = \frac{1}{1}$

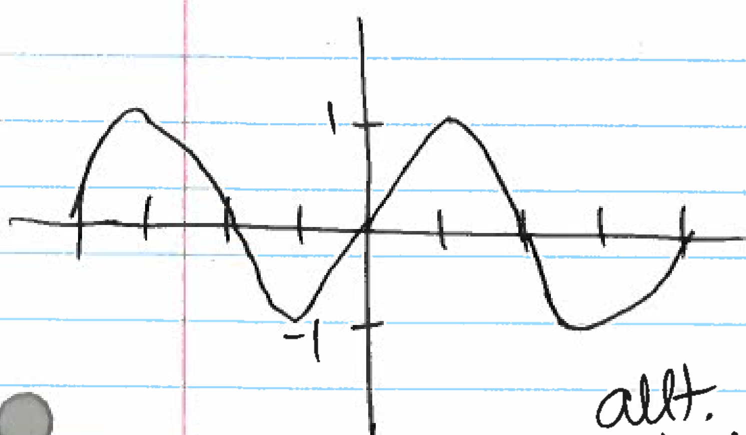
Day 2

Unit Circle (0,1)



4.4 Graphs of Sine & Cosine: Sinusoids

Basic Function: The sine Function



$f(x) = \sin x$

Domain $(-\infty, \infty)$

Range $[-1, 1]$

Continuous

alt. increasing & decreasing → Symmetric WRT to origin (odd)

Bounded limits

ab max 1

ab min -1

NO HA

NO VA

End beh.

$x \to -\infty$

$x \to \infty$

$x \to \infty$

$x \to \infty$

Period $= 2\pi$ (Repeats at each period)

Determing the period of a trig function of sine or cosine

$$\boxed{P = \frac{2\pi}{B}}$$

FORMULA

ex $f(x) = \sin x$

(location of B in front of the X)

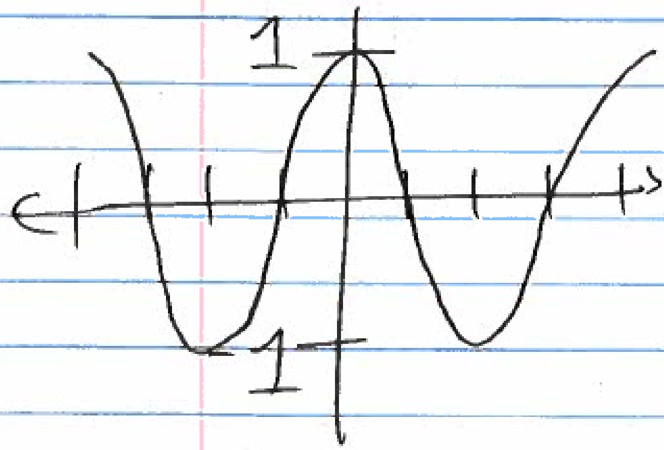
in this case it is $B = 1$

$$P = \frac{2\pi}{1} = 2\pi$$

Period

more later...

Basic Function: The Cosine Function



$f(x) = \cos x$

Domain $(-\infty, \infty)$

Range $[-1, 1]$

Continuous

Alt increasing ; decreasing

Symmetry WRT y-axis (even)

Bounded

Ab max = 1

ab min = −1

NO HA

NA VA

End behavior:

$\lim_{x \to -\infty} f(x) \to \infty$

$\lim_{x \to \infty} f(x) - \infty$

Period = 2π

Repeats at each period

$$\boxed{P = \frac{2\pi}{|B|}}$$

$f(x) = \cos x$

B-value

1/4 X

$$P = \frac{2\pi}{1} = \underline{2\pi}$$

Definition: Amplitude of Sinusoid

ex $f(x) = a \sin(bx + c)$

fraction

greater than 1

a - amplitude (shrink or stretch)

B - for period formula

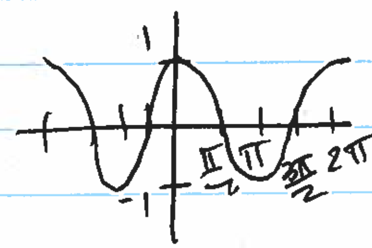
C - shift up or down

$\boxed{P1}$ Vertical Stretch or Shrink & Amplitude

$\boxed{ie1}$ $y_1 = \cos x$

amplitude $= 1 : -1$

$B = 1$



$p = \frac{2\pi}{|B|}$

$p = 2\pi$

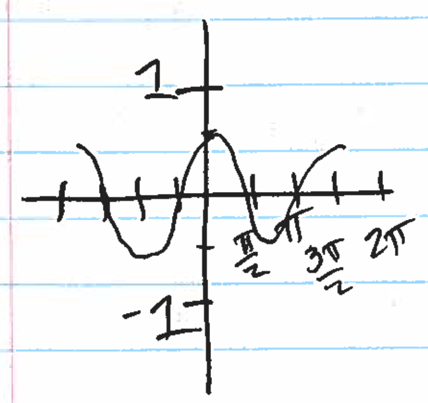
$2\pi \cdot \frac{1}{4} : \frac{\pi}{2}$

$2\pi \cdot \frac{2}{4} : \pi$

$\boxed{ie2}$ $y_2 = \frac{1}{2} \cancel{*} (\cos x)$

$2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$

$2\pi \cdot \frac{4}{4} = 2\pi$



amplitude $= \frac{1}{2} \; ; -\frac{1}{2}$

Vertical shrink of ½

went from $-1 : 1$ as your limit to $\frac{1}{2} : -\frac{1}{2}$ as the limit

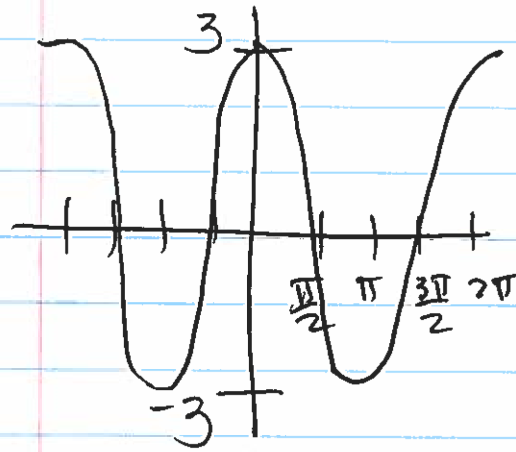
$p = \frac{2\pi}{|B|}$ $\quad B = 1$

$\frac{2\pi}{1} = 2\pi$

5

c) $y_3 = -3\cos x$

B



Vertical stretch of 3

limit now -3, & $+3$

$p = \frac{2\pi}{|B|}$ $B = 1$

$= \frac{2\pi}{1} = 2\pi$

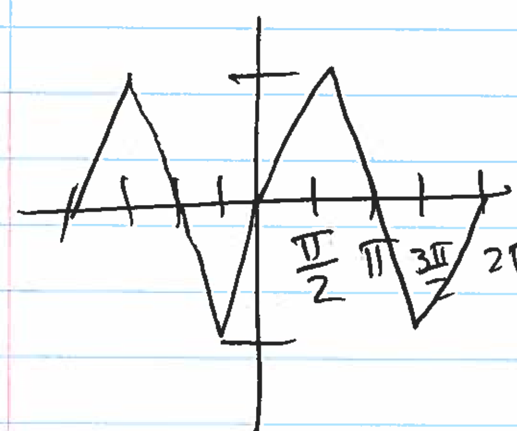
Period of Sinusoid (sine)

ie 1) $y_1 = \sin x$

$B = 1$

amplitude $= 1$

limit -1 & 1



$p = \frac{2\pi}{|B|} = \frac{2\pi}{1}$

$2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$ $2\pi \cdot \frac{2}{4} = \pi$

$2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$ $2\pi \cdot \frac{4}{4} =$

ie 2 $y = -2\sin\left(\frac{x}{3}\right)$ Find A : B : Period

$y = -2\sin\frac{1}{3}(x)$
$\quad\; a \quad\;\; B$

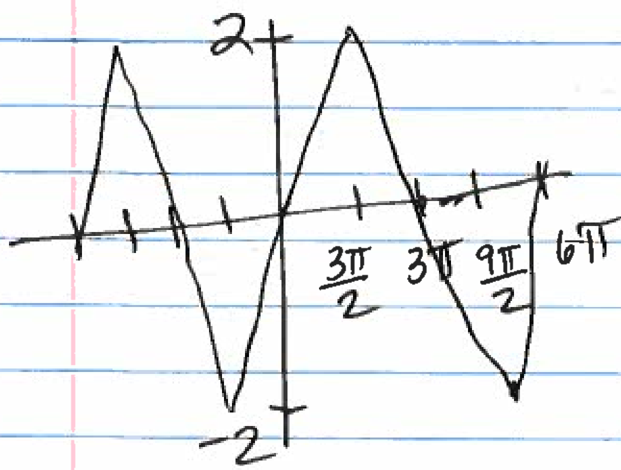
amplitude $= 2$

limit -2 | $+2$

Period $= \frac{2\pi}{B}$

$B = \frac{1}{3}$

so... $\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$



* Now find the correct marks when period not 2π

4 Quarters think unit circle

$6\pi \cdot \left(\frac{1}{4}\right) = \frac{6\pi}{4} = \frac{3\pi}{2}$

$6\pi \cdot \left(\frac{2}{4}\right)$ 3π

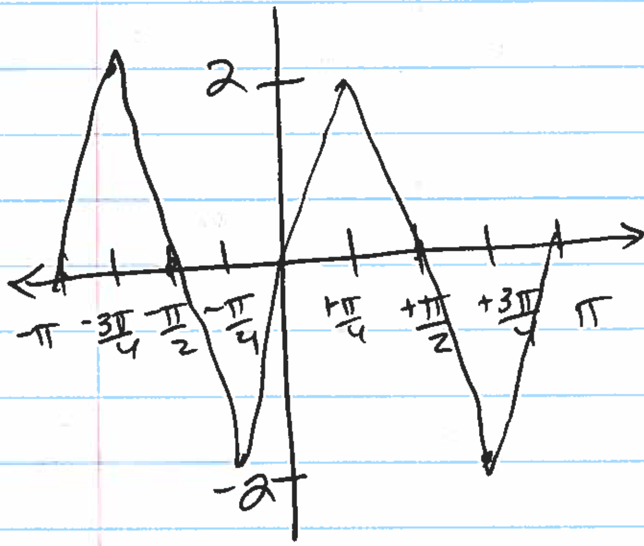
* opposites sign on negative side of x

$6\pi \cdot \left(\frac{3}{4}\right) = \frac{18\pi}{4}$ $\frac{9\pi}{2}$

$6\pi \cdot \left(\frac{4}{4}\right) = 6\pi$

7

ie3 $y = 3\sin(-2x)$



Amplitude: 3

$B = -2$

$P = \frac{2\pi}{B} = \frac{2\pi}{|-2|} = \frac{2\pi}{2} = \pi$

Find $\frac{1}{4}$'s

$\pi \cdot \frac{1}{4} = \frac{\pi}{4}$ $\pi \cdot \frac{2}{4} = \frac{\pi}{2}$

$\pi \cdot \frac{3}{4} = \frac{3\pi}{4}$ $\pi \cdot \frac{4}{4} = \pi$

P24 $f(x) = 4\sin\left(2x/3\right)$

$f(x) = 4\sin\frac{2}{3}(x)$

$A = 4 \,\vdots\, -4$

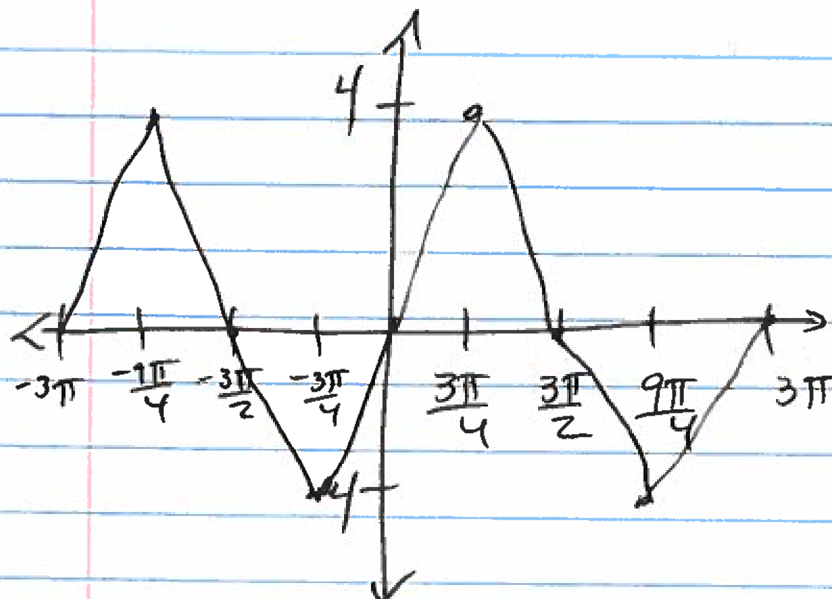
$B = \frac{2}{3}$

$P = \frac{2\pi}{|B|}$

$P = \frac{2\pi}{\frac{2}{3}}$

$P = \cancel{2}\pi \cdot \frac{3}{\cancel{2}}$

$P = 3\pi$



also called frequency of function

Intervals $\left(\frac{1}{4}\text{'s}\right)$

$3\pi \cdot \frac{1}{4} = \pm\frac{3\pi}{4}$ $\qquad$ $3\pi \cdot \frac{2}{4} = \pm\frac{3\pi}{2}$

$3\pi \cdot \frac{3}{4} = \pm\frac{9\pi}{4}$ $\qquad$ $3\pi \cdot \frac{4}{4} = \pm 3\pi$

through #17

do # 13 in book

Very important!

do not
skip this

these are for
13-33 odds

(cover phase changes)

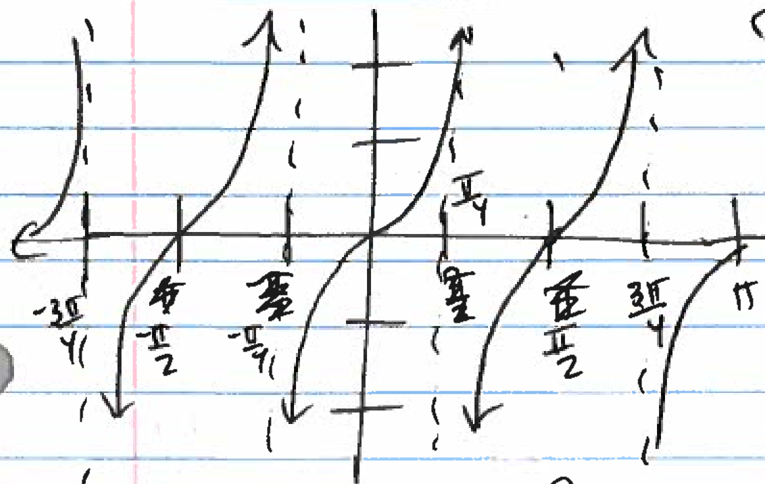
4.5 Tangent, Cotangent, Secant, & Cosecant

Intervals $\frac{\pi}{|B|}$ π

$\pm\frac{\pi}{4}$; $\frac{\pi}{2}$; $\frac{3\pi}{4}$, π

The Tangent Function

$f(x) = \tan x$



1 space for VA!

Domain:
All reals except odd multiples of $\frac{\pi}{2}$

Range: all reals

Continuous (on its domain)

Increasing on each interval

Symmetric WRT origin (odd)

Not bounded above or below

No extremas

NO HA

$VA = k \cdot \left(\frac{\pi}{2}\right)$

End Behavior

$\lim_{x \to -\infty} = -\infty$

$\lim_{x \to \infty} = \infty$

1

Very Important:

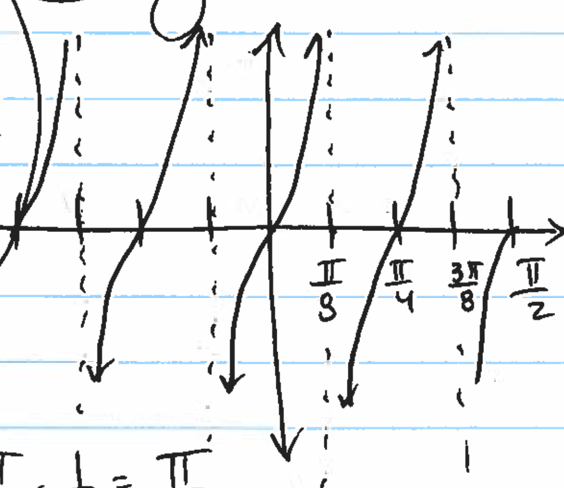
$$\tan x = \frac{\sin x}{\cos x} = \left(\frac{y}{x}\right)$$ Remember

P1 Graphing Tangent Function

has asymptotes at the zeros of cosin

Amplitude: does not exist

ie1 $y = +\tan 2(x)$



Shrinking by $\frac{1}{2}$

$\frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$

V.A located here

$B = 2$

$$\boxed{\text{FORMULA Period} = \frac{\pi}{|B|}} = \frac{\pi}{2}$$

$\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$

$\frac{\pi}{2} \cdot \frac{2}{4} = \frac{\pi}{4}$

$\frac{\pi}{2} \cdot \frac{3}{4} = \frac{3\pi}{8}$

Intervals:

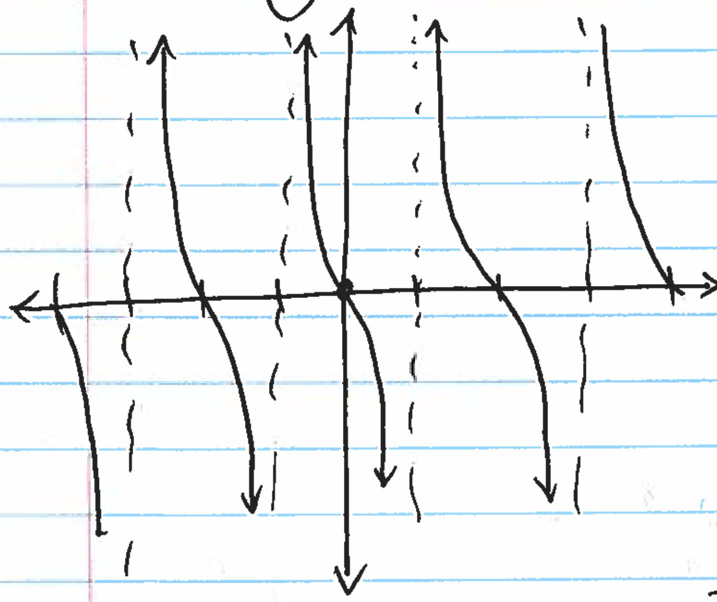
$\pm\frac{\pi}{8}$; $\pm\frac{\pi}{4}$; $\pm\frac{3\pi}{8}$

$\pm\frac{\pi}{2}$

* Crosses the origin & skips

negative reverses the curv

$\boxed{\text{ie 2}}$ $y = -\tan 2x$



Amp: does NOT exist

$p = \frac{\pi}{|B|} = \frac{\pi}{2}$

$B = 2$

Intervals:

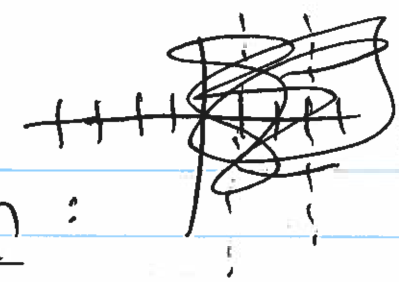
$\pm\frac{\pi}{8};\ \pm\frac{\pi}{4};\ \pm\frac{3\pi}{8};\ \frac{\pi}{2}$

$\frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$

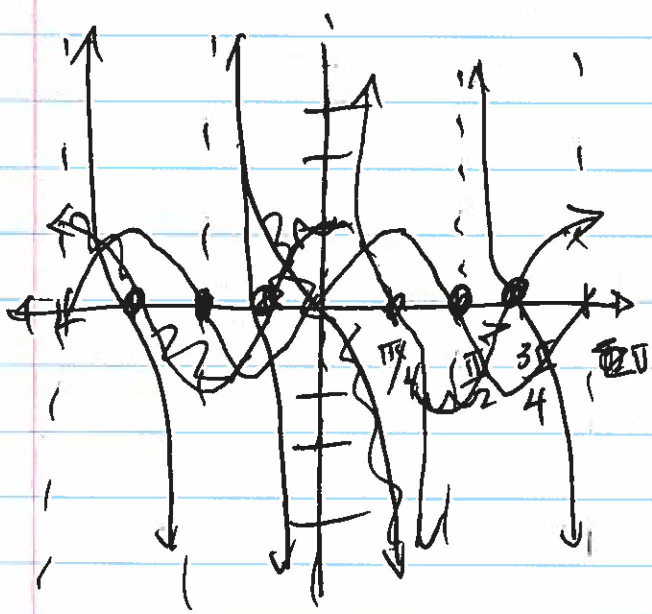
$\frac{\pi}{\cancel{2}}\cdot\frac{\cancel{2}}{4} = \frac{\pi}{4}$ *

$\frac{\pi}{2}\cdot\frac{3}{4} = \frac{3\pi}{8}$

The Cotangent Function :



$\cot x = \dfrac{\cos x}{\sin x} = \left(\dfrac{x}{y}\right)$ ← know this



$f(x) = \cot x$

Amp: d/N/E

Period $= \dfrac{\pi}{|B|}$

$P = \pi$

* Zeros at the ~~cosine~~ zeros sine function

$\pi \cdot \frac{1}{4} = \frac{\pi}{4}$

* The Cotangent has asymptotes at the Zeros of sine function

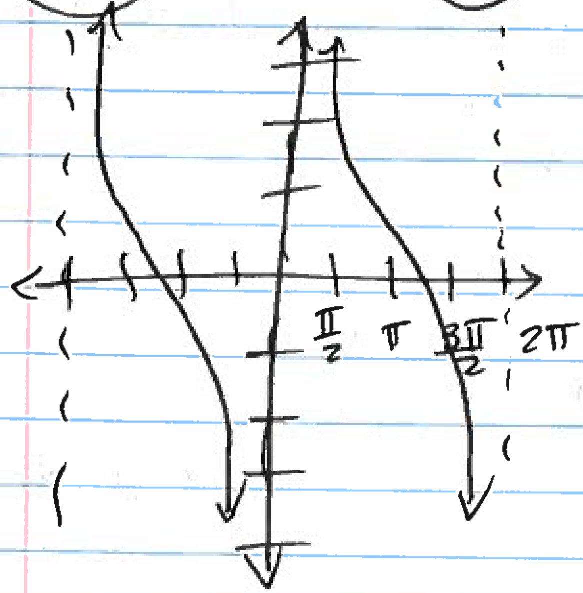
$\pi \cdot \frac{1}{2} = \frac{\pi}{2}$

$\pi \cdot \frac{3}{4} = \frac{3\pi}{4}$

* the cotant has Zero @ the Zeros of the cosine function

$\boxed{P2}$ Graphing a Cotangent Function

ie 1 $f(x) = 3 \cot\left(\frac{x}{2}\right) + 1$ ← use calculator



$f(x) = 3 \cot \frac{1}{2}(x) + 1$

Amp. NONE

$P = \frac{\pi}{|B|} = \frac{\pi}{\frac{1}{2}} = 2\pi$

$2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$

$2\pi \cdot \frac{2}{4} = \pi$

$2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$

2π

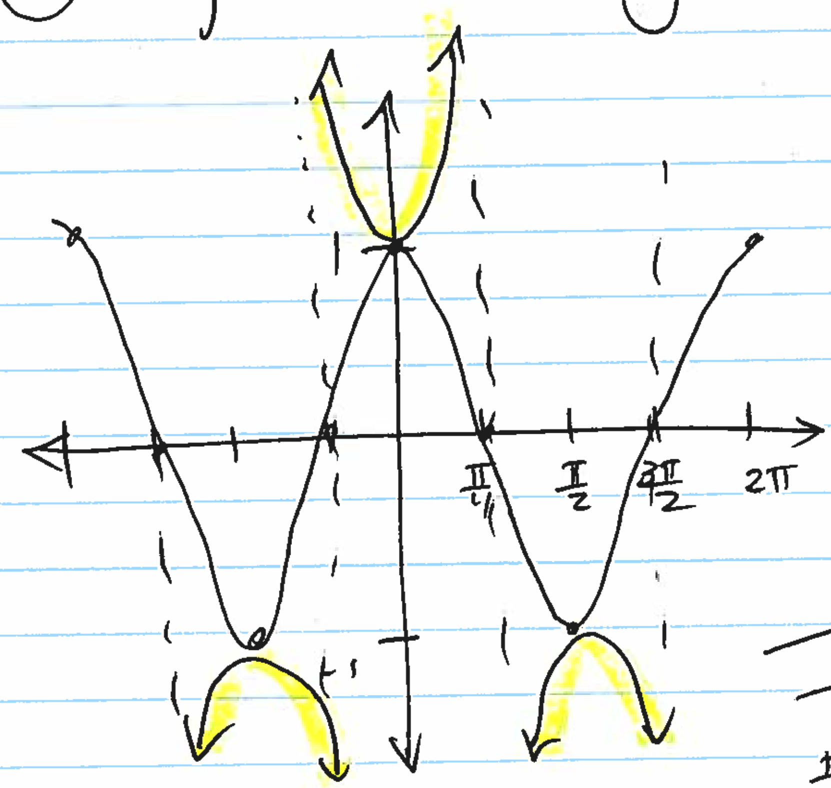
Secant Function:

* Reciprocal of ~~sin~~ cosine function

① Graph Cosine first

② graph secant second off of max & min of cosin

ex $y = \cos(x)$ & $y = \sec x$



Amplitude:

1 & -1

$B = 1$

Period $= \frac{2\pi}{|B|}$

$= 2\pi$

Interval

$\pm\frac{\pi}{4}$; $\frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

$\boxed{P3}$ Solving Trig. Equations With Algebra

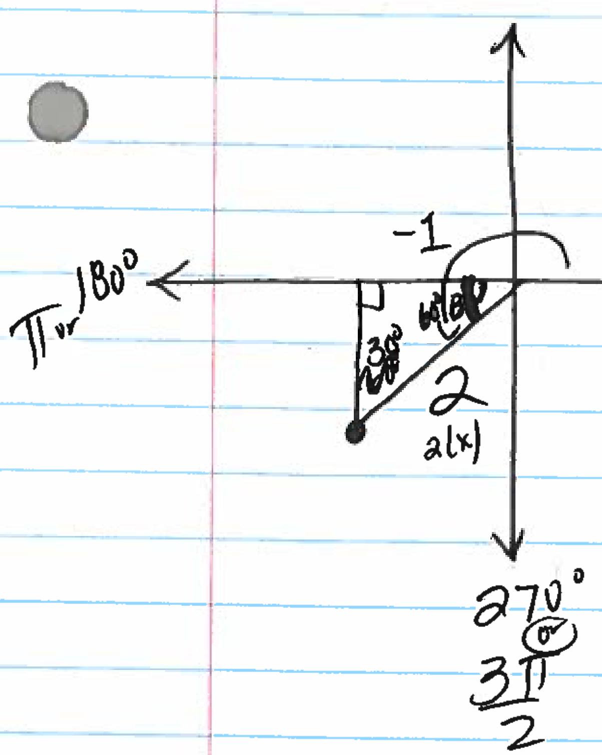
ie1 Find the value of x b/t π ; $\frac{3\pi}{2}$ that solves

$\sec x = -2$ → opposite is → $\cos x = -\frac{1}{2}$

$\cos \Rightarrow x$-value

Angles: π ; $\frac{3\pi}{2}$

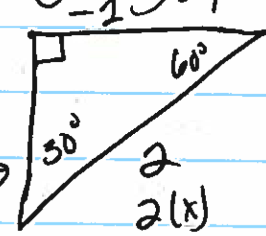
180 & 270



$180 + 60 = 240°$ or → $\frac{4\pi}{3}$ radian

$\cos x = \frac{-1}{2}$ $\frac{A}{H}$

Recognize (x)a, 30°, 60°, 90° △



find 30° by using formula

Through #29 Hull

9

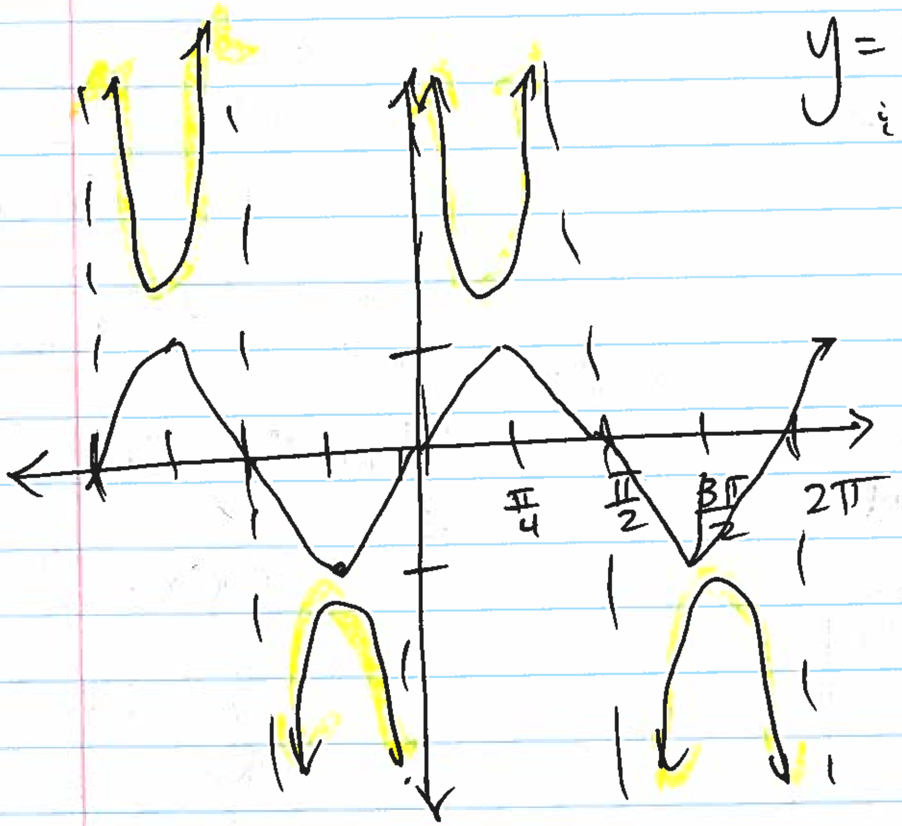
69.)

The Cosecant Function

opposite is sine function

① graph sine
② Find csc where extremas are located

$y = \sin(x)$ & $y = \csc(x)$



mustdo!

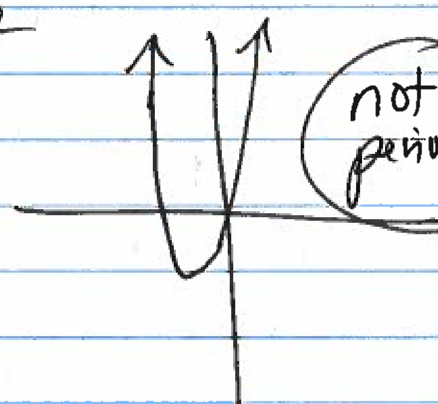
#37 FROM HWK

4.6 Graphs of Composite Trigonometric functions

$\boxed{P1}$ Combining the Sine Function with x^2; Which appears to be periodic?

Vertical window setting $-2\pi \le x \le 2\pi$

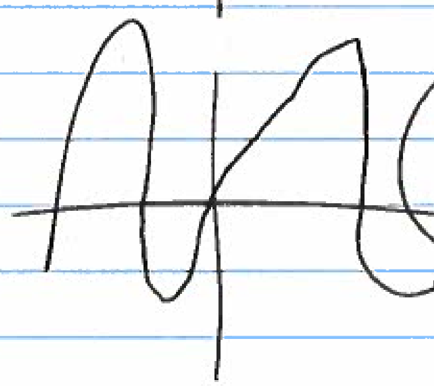
a) $y = \sin x + x^2$



not periodic

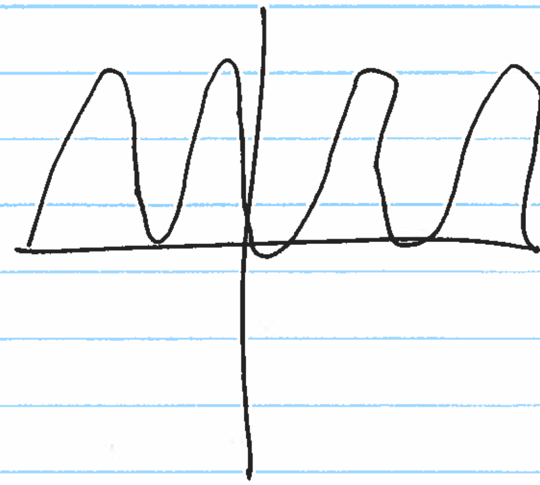
Periodic: Repeats at end of period

b) $y = x^2 \sin x$



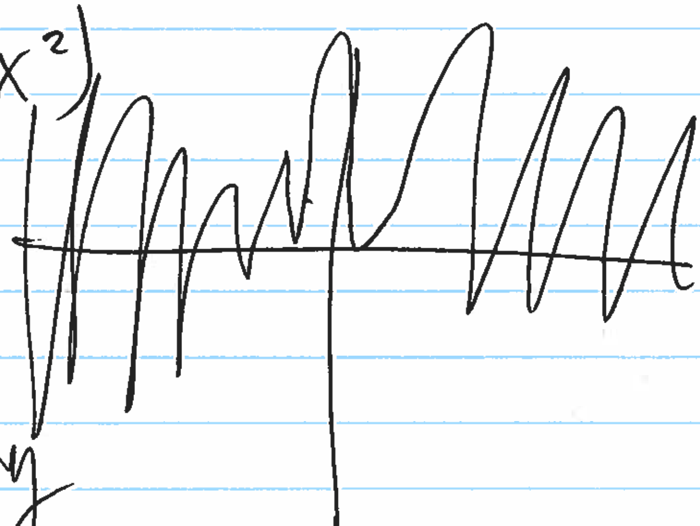
not periodic

c.) $y = (\sin x)^2$



yes, repeat pattern

d) $y = \sin(x^2)$



not periodic

not repeating

? P2 Verifying Periodicity Algebraically

ie1 $f(x) = (\sin x)^2$ period of sine $= 2\pi$

$f(x+2\pi) = (\sin(x+2\pi))^2 \sin(x+2\pi)$

$= (\sin(x))^2$

$= f(x)$

P3 Composing $y = \sin x$; $y = x^3$

ie1 $f(x) = \sin^3 x$ sine period $= 2\pi$

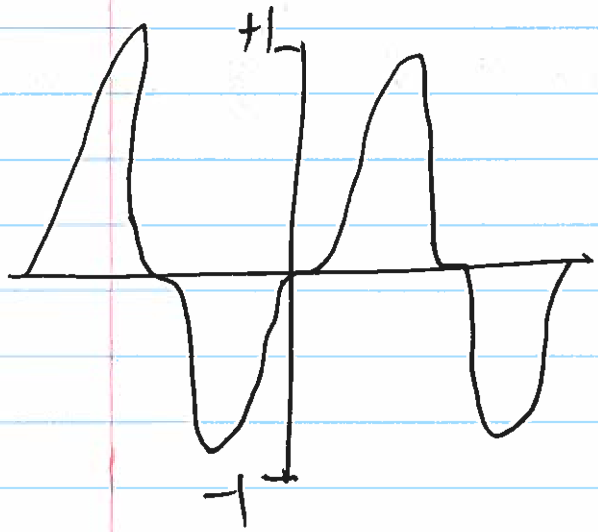
$f(x+2\pi) = \sin^3(x+2\pi)$

$= (\sin(x+2\pi))^3$ changing notation

$= (\sin(x))^3$ period of sine

$= \sin^3(x)$ changing notation

$= f(x)$ 3



$D: (-\infty, \infty)$

$R: [-1, -1]$

P4 Analyzing Nonnegative Periodic Functions

Find domain, range, and period.
Sketch the graph showing 4 periods

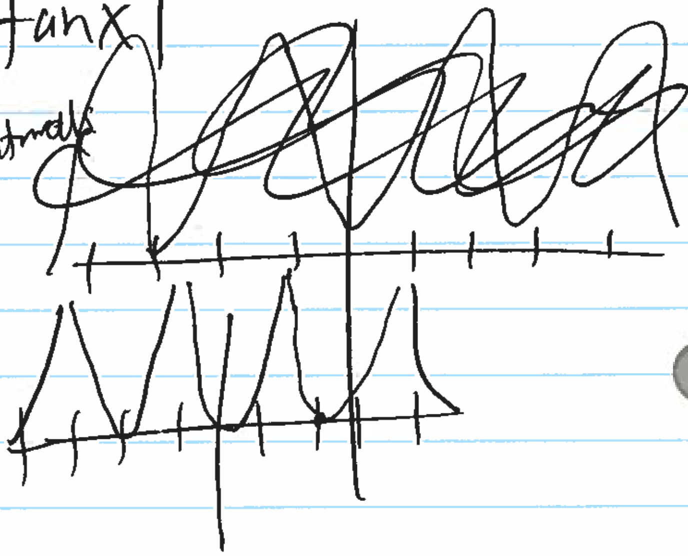
1.) $f(x) = |\tan x|$

D: all reals except $\frac{\pi}{2}$ intervals

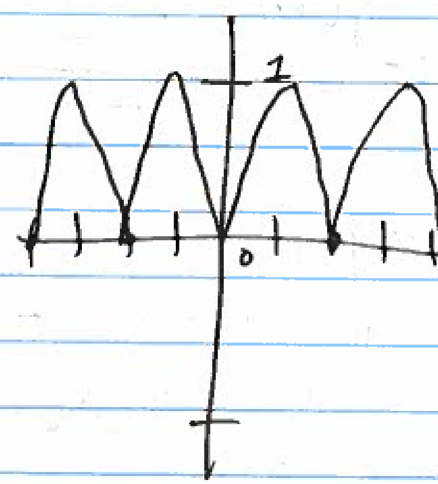
R: ( )

$P = \frac{\pi}{|B|}\ \frac{\pi}{1} = \pi$

Intervals $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$



b) $g(x) = |\sin x|$



D: $(-\infty, \infty)$

R: $[0, 1]$

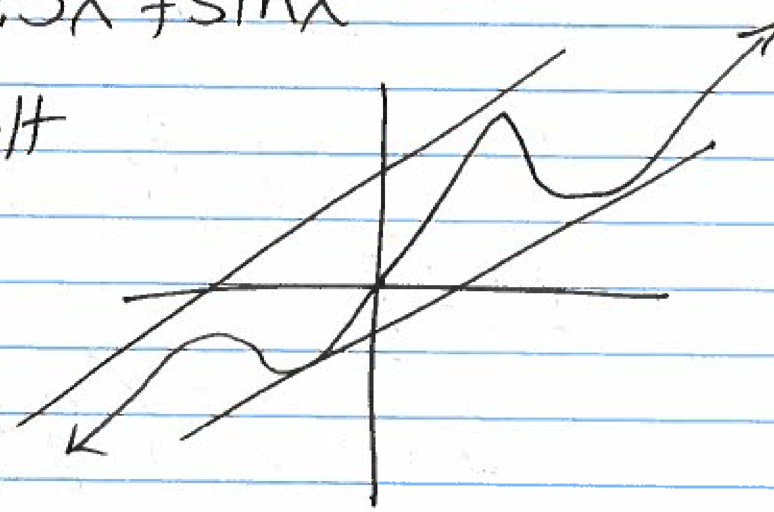
Period $= \frac{2\pi}{|B|} = 2\pi$

Intervals: $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

P5 Adding a Sinusoid to a Linear Function.

ie1 $f(x) = 0.5x + \sin x$

* Oscillated b/t 2 parallel lines



Sums & Diffrence of Sinusoids

from 4.4 → $\boxed{y_1 = a_1 \sin(b(x-h_1)) + K}$

$\boxed{y_2 = a_2 \cos(b(x-h_2)) + \cancel{K}}$

$$y_1 + y_2 = a_1 \sin((b(x-h_1)) + K + a_2 \cos(b(x-h_2)) + K$$

$\boxed{P6}$ Identify a Sinusoid

1.) $f(x) = \underset{a}{5} \cos_{b} x + \underset{a}{3} \sin_{b} x$

$\underline{\cos}\ \frac{2\pi}{B}$ $\quad$ $\sin \frac{2\pi}{B}$

$\frac{2\pi}{1} = 2\pi$ $\quad$ $y\cancel{o}$ $\quad$ $\frac{2\pi}{1} = 2\pi$

2) $\cos 5x + \sin 3x$

$\cos = \frac{2\pi}{5} = \frac{2\pi}{5}$ $\sin = \frac{2\pi}{3}$ NO

3) $2\cos 3x$ $3\cos 2x$

$\frac{2\pi}{3}$ NO $\frac{2\pi}{2} = \pi$

4) $a \cos \frac{3}{7}x$ $\frac{3}{7}x$ $\sin \frac{3}{7}x$

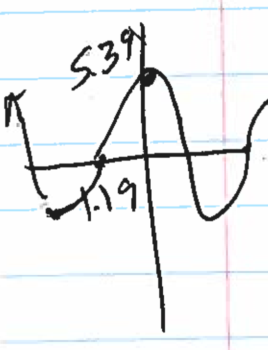
$\frac{2\pi}{\frac{3}{7}}$ $\frac{2\pi}{\frac{3}{7}}$ $\frac{2\pi}{3/7}$

$\frac{14\pi}{3}$ $\frac{14\pi}{3}$ $\frac{14\pi}{3}$

Yes

P7 Expressing the Sum of Sinusoids as a Sinusoid

ic1 $f(x) = 2\sin x + 5\cos x$



a) Find period $\frac{2\pi}{1} = 2\pi$

b) Estimate the amplitude . . .

5.39 maximum value
1.19 x-intercept closest to ☺

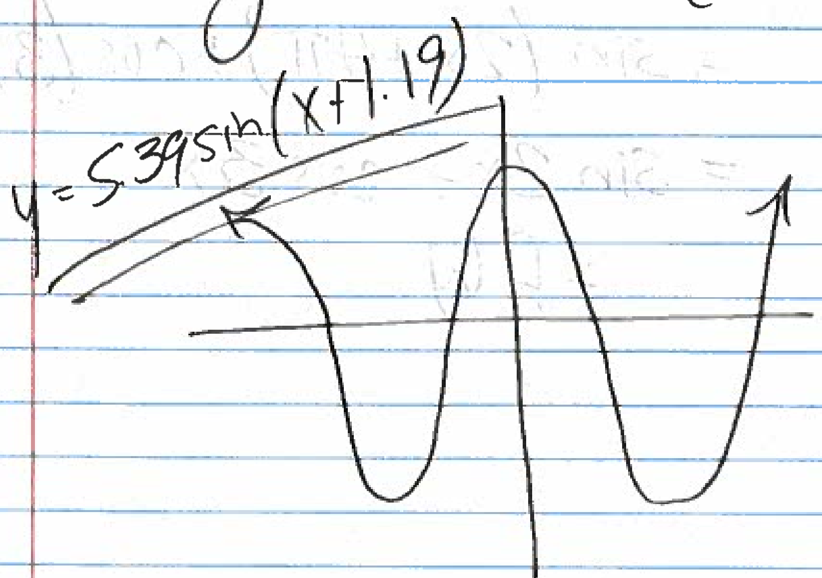
$f(x) = a\sin(x+h)$

$\approx 5.39\sin(x+1.19)$

c) Given the sinusoid $a \sin(b(x-h))$ that approximats $f(x)$

$$y = 2\sin x + 5\cos x$$

$$y = 5.39 \sin(x + 1.149)$$



graphs virtually indentical

P8 Showing a Function is Periodic but not Sinusoid

1c2 $f(x) = \sin 2x + \cos 3x$

$f(x+2\pi) = \sin(2(x+2\pi))$

$= \sin(2x+4\pi) + \cos(3x+6\pi)$

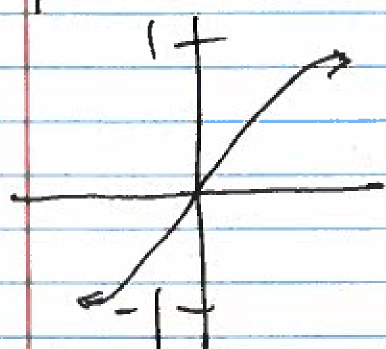
$= \sin 2x + \cos 3x$

$= f(x)$

4.7 Inverse Trig. Functions

Inverse sine function: $y = \sin^{-1} x$ (called arcsine of x)

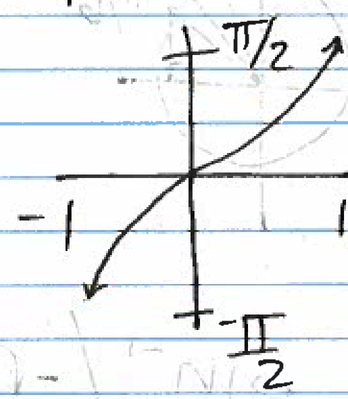
$y = \sin x$



$D: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

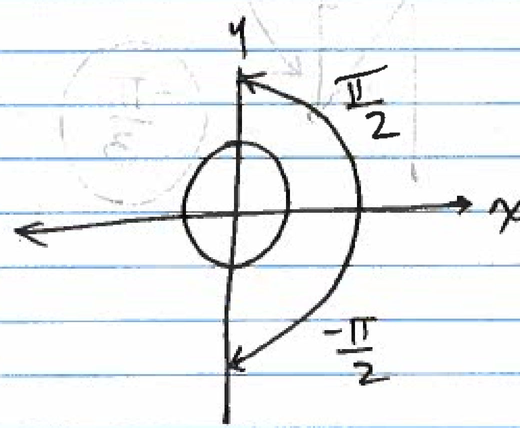
$R: [-1, 1]$

$y = \sin^{-1} x$



$D: [-1, 1]$

$R: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



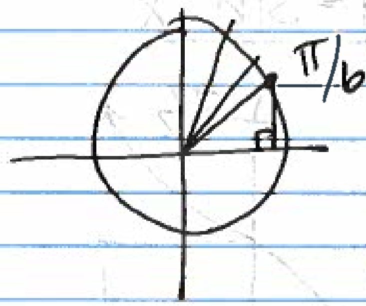
* Values of $\sin^{-1} x$ will always be found on the right side of the unit circle

①

P1 Evaluating $\sin^{-1}x$ With a calc

Find exact value w/out calculator.

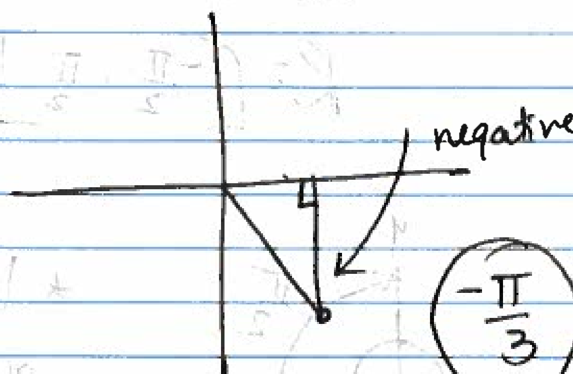
1.) $\sin^{-1}\left(\frac{1}{2}\right)$



Sin = y-value

2) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

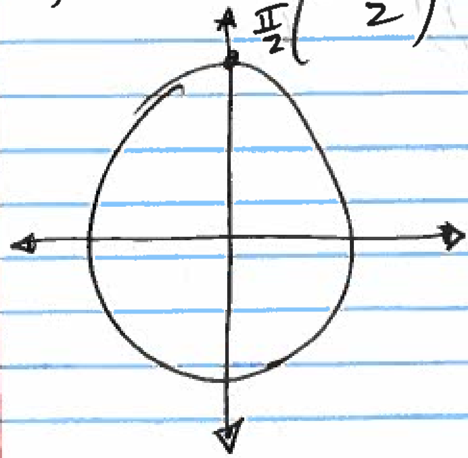
sine = y-value



$-\frac{\pi}{3}$

3.) $\sin^{-1}\left(\frac{\pi}{2}\right)$

sine = y-value

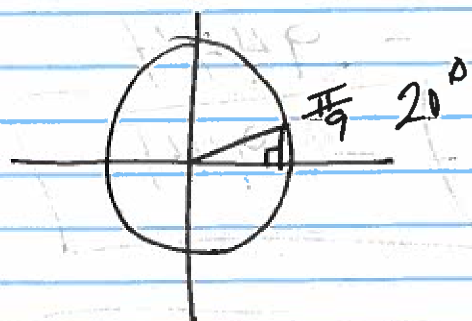


does not exist

$D: [-1, 1]$

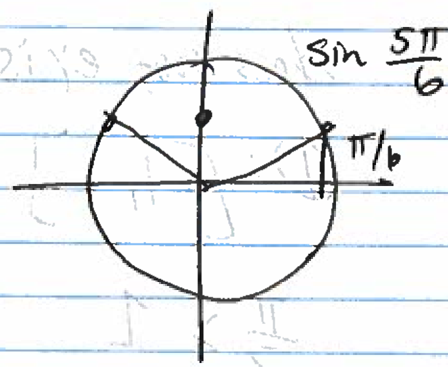
$\frac{\pi}{2} > 1$

4.) $\sin^{-1}\left(\sin\left(\frac{\pi}{9}\right)\right) = \frac{\pi}{9}$



5.) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \frac{\pi}{6}$

150°



P2 Evaluating $\sin^{-1}$ with a Calculator

1.) $\sin^{-1}(-0.81) = -.9444$

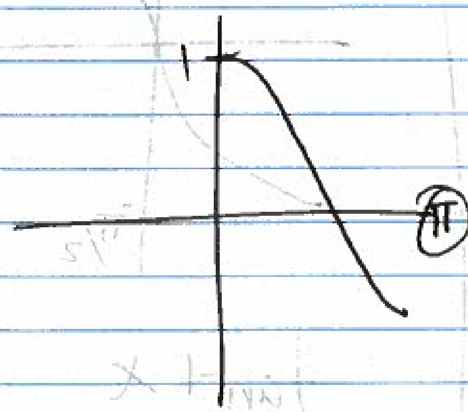
$= \boxed{-.944}$

2.) $\sin^{-1}(\sin(3.49\pi)) = \boxed{-1.54}$

also calld ~~arcsine~~ Arccosine of x

Inverse Cosine & Tangent Functions

$y = \cos^{-1} x$

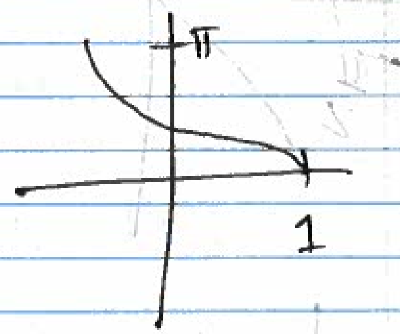


$y = \cos x$

$D: [0, \pi)$

$R: [-1, 1]$

* These visual



$y = \cos^{-1} x$

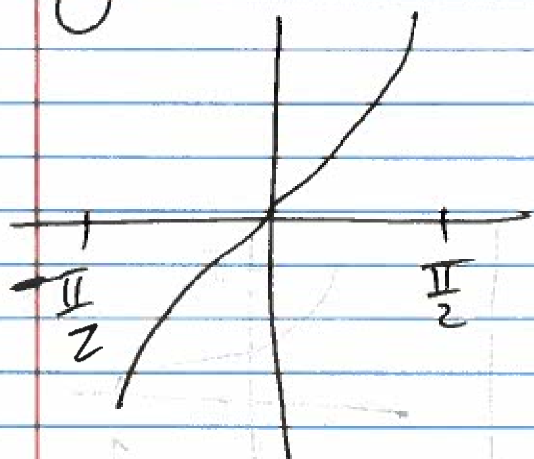
$D: [-1, 1]$

$R: (0, \pi)$

These visual

Also called arctangent

$y = \tan x$

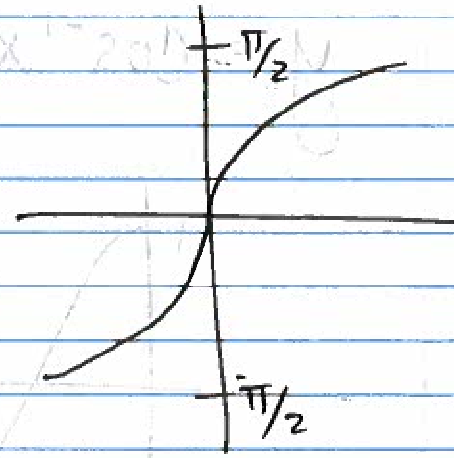


$\underline{\tan x}$

$D: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$R: (-\infty, \infty)$

$y = \tan^{-1} x$



$\underline{\tan^{-1} x}$

$D: (-\infty, \infty)$

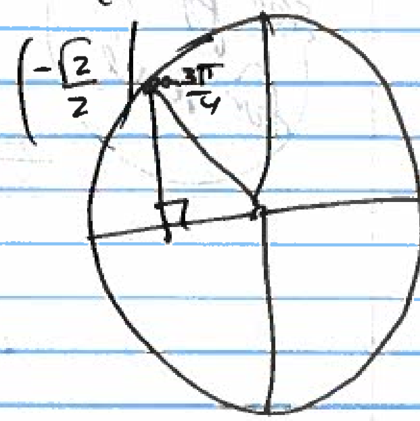
$R: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Flipped

P3 Evaluating Inverse Trig Functions w/out a calculator (use unit circle)

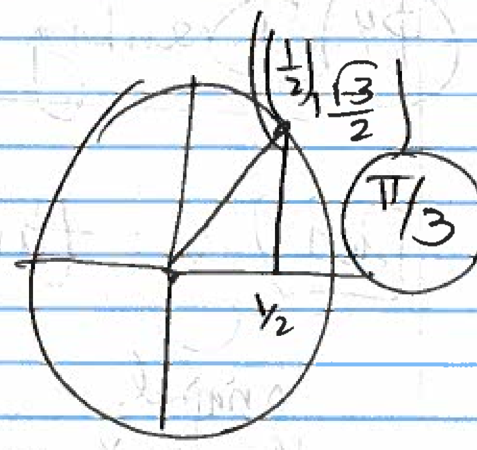
cos -> x-value

a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$



$= \frac{3\pi}{4}$

b) $\tan^{-1}(\sqrt{3})$



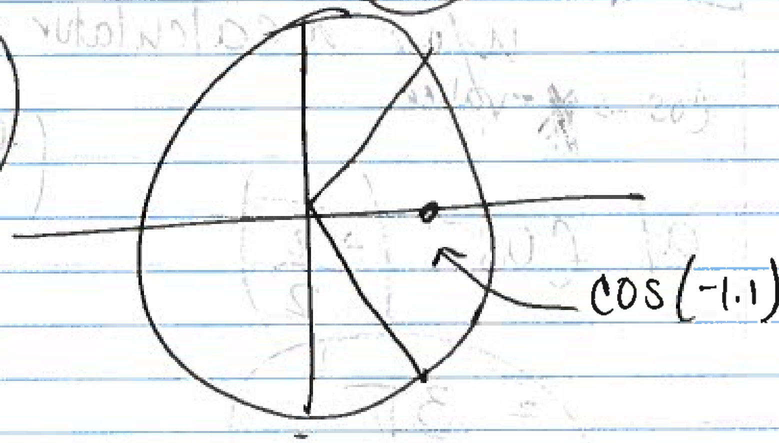
$\tan = \frac{y}{x}$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3}$$

$= \frac{\pi}{3}$

~~c)~~ yuck! $\cos^{-1}(\cos(-1.1)) = 1.1$

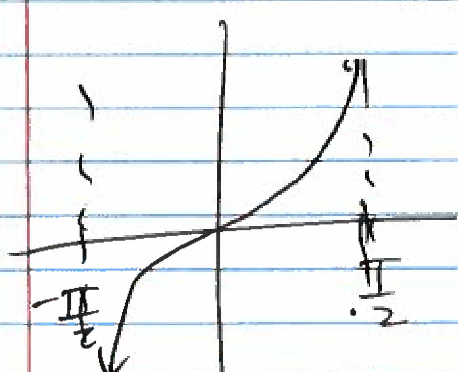
see pg. 9 for rule



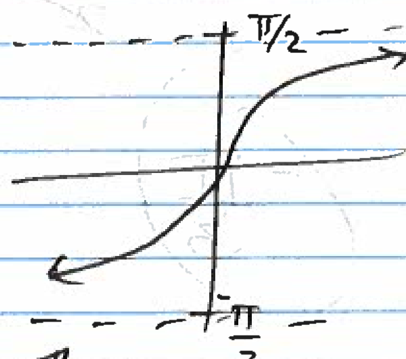
p4 Describing End Behavior

$\boxed{\text{ie1}}$ $y = \tan^{-1} x$

original
$y = \tan x$



$y = \tan^{-1} x$



$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

$\lim_{x \to -\infty} \tan^{-1} x = \frac{\pi}{2}$

Rules

Composing Trig. & Inverse Trig. Functions

Always true:

$\sin(\sin^{-1}(x)) = X$

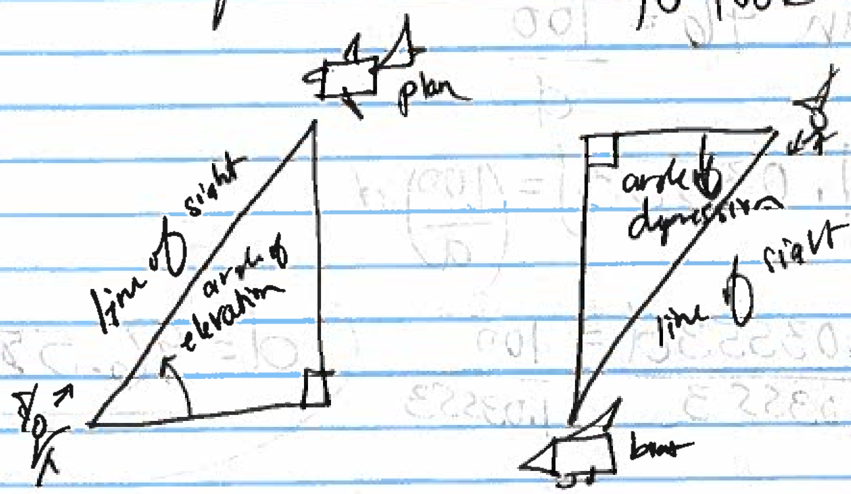
\# $\cos(\cos^{-1}(x) = X$

$\tan(\tan^{-1}(x) = (X)$

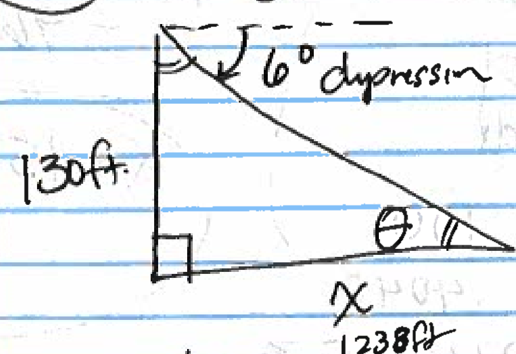
4.8 Solving Problems with Trig.

Angle of Elevation: eye moves up to look at something

Angle of depression: eyes move down to look at something



P1 Using Angle of Depression



Find X

$\tan\theta = \frac{130}{x}$

$x\left(\tan 6^\circ\right) = \left(\frac{130}{x}\right)x$

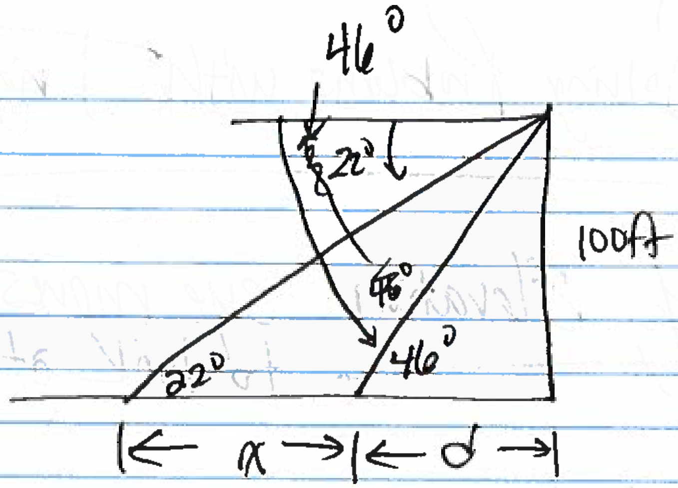
$(\tan 6^\circ)(x) = 130$

$\frac{.105(x)}{.105} = \frac{130}{.105}$

1238 ft

* Alternate interior Angle

ie2



$$\tan 46 = \frac{100}{d}$$

$$d(1.03553) = \left(\frac{100}{d}\right) d$$

$$\frac{1.03553d}{1.03553} = \frac{100}{1.03553}$$

$d = 96.58$ ft

$$\tan 22 = \frac{100}{x+d}$$

$$x+d\,(.4040) = \left(\frac{100}{x+d}\right)$$

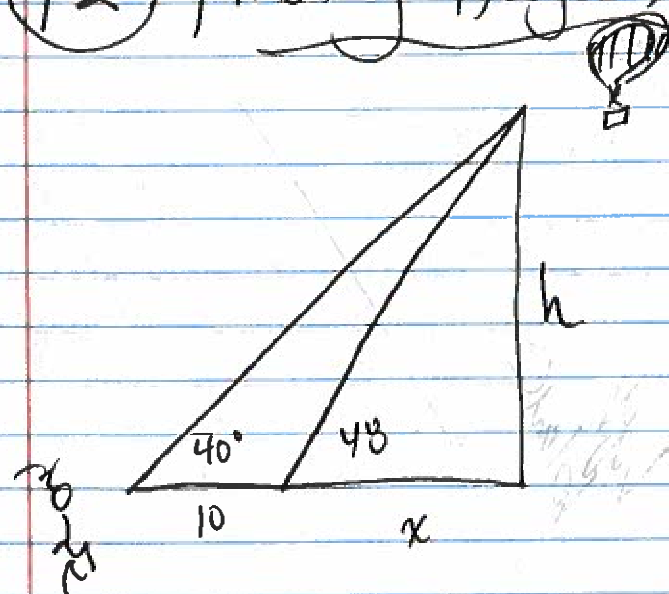
$$\frac{.4040(x+d)}{.4040} = \frac{100}{.4040}$$

$$x+d = 247.53$$
$$-d \qquad -d$$

$$x = 247.53$$
$$-96.58$$

$x = 150.94$ ft

P2 Finding Height Above Ground



Solve for $\underline{\underline{h}}$

$$x\left(\tan 48 = \frac{h}{x}\right)x \qquad h = (\tan 48)\,x$$

$$10+x\left(\tan 40^\circ\right) = \left(\frac{h}{10+x}\right)10+x \qquad h = \tan 40(10+x)$$

$\underline{\underline{\tan 48^\circ}}$ $\tan 40^\circ$

$$h = \underline{1.1106}\,(x) \qquad h = .8391\,(10+x)$$

$$h = 1.1106x \qquad h = \underline{\underline{8.391 + .8391x}}$$

$\underline{\underline{\text{substitution}}}$

$$\begin{array}{r} 1.1106x = 8.391 + .8391x \\ \underline{-.8391x \qquad\qquad -.8391x} \\ \frac{.2715x}{.2715} = \frac{8.391}{.2715} \end{array} \qquad x = \underline{\underline{30.9}}$$

Left Blank

$X = 30.9$ ft

Find $h = 1.1106(x)$

$h = 1.1106(30.9)$

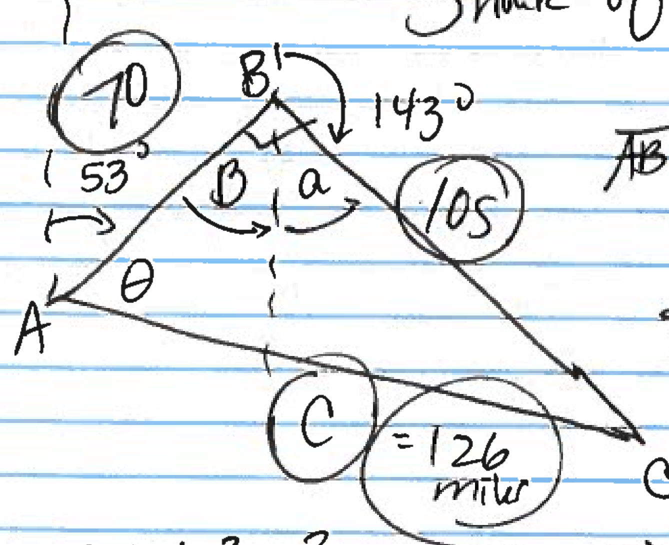
$h = 34.32$ ft

P3 Using Trig. in Navigation

pg 427 Port Cleveland 35 Knot

2 hours of 53°

3 hours of 143°



AB $35 \times 2 = 70$ naut miles

BC $35 \times 3 = 105$ naut miles

$a^2 + b^2 = c^2$

$70^2 + 105^2 = c^2$

$4900 + 11025 = c^2$

$15925 = c^2$

$c = 126.2$