7.1 Solving Systems of 2 Equations

$\boxed{P1}$ Using Substitution Method to find the solution / answer

(ic1) $2x - y = 10$ ①

$3x + 2y = 1$ ②

$(3, {}^-4)$

① write #1 as $y=$

$$2x - y = 10$$
$$-2x \quad -2x$$
$$\frac{-1y}{-1} = \frac{-2x}{-1} + \frac{10}{-1}$$

$$y = 2x - 10$$

② Subst. into #2 : solve for x

$$3x + 2y = 1$$
$$3x + 2(2x - 10) = 1$$
$$3x + 4x - 20 = 1$$
$$7x - 20 = 1$$
$$+20 \quad +20$$
$$\cancel{3}7x = 21$$
$$-7 \quad 3$$

$x = 3$

①

③ Subst $x=3$ into #1

$$2x - y = 10$$
$$2(3) - y = 10$$
$$6 - y = 10$$
$$-6 \quad -6$$
$$\frac{-y}{-1} = \frac{4}{-1}$$

$y = 4$

P2 Solving a Nonlinear System by Substitution

ie1 Rectangular Gardn perimetr 100ft
& Area of 300 ft^2

perimetr $\quad 2x + 2y = 100 \quad$ ①

Area $\quad x \cdot y = 300 \quad$ ②

① Write #1 as $y =$

$$2x + 2y = 100$$
$$-2x \qquad -2x$$
$$\frac{2y}{2} = \frac{-2x}{2} + \frac{100}{2}$$
$$y = -1x + 50$$

② Subst $y =$ into 2nd & solve for x

$$x \cdot y = 300$$
$$x(-1x + 50) = 300$$
$$-x^2 + 50x = 300$$
$$\qquad -300 \quad -300$$
$$-x^2 + 50x - 300 = 0$$

Graph to find X where $y = 0$

$x = 6.97$

$x = 43.03$

③ Subst X values into ① & solve for y

$2x + 2y = 100$

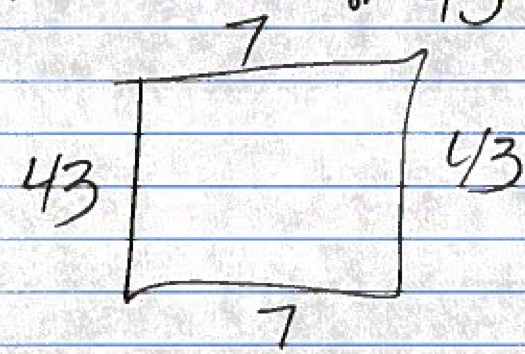
$2(6.97) + 2y = 100$ $\qquad$ $2(43.03) + 2y = 100$

$y = 43.027$ $\qquad$ $y = 6.972$

$(6.97, 43.027)$ $\qquad$ $(43.03, 6.972)$

7×43 $\qquad$ 43×7

Rectangle is 7×43 or 43×7



3

2

P3 Solving a Non-linear System Algebraically

ie 1 $y = x^3 - 6x$

$y = 3x$

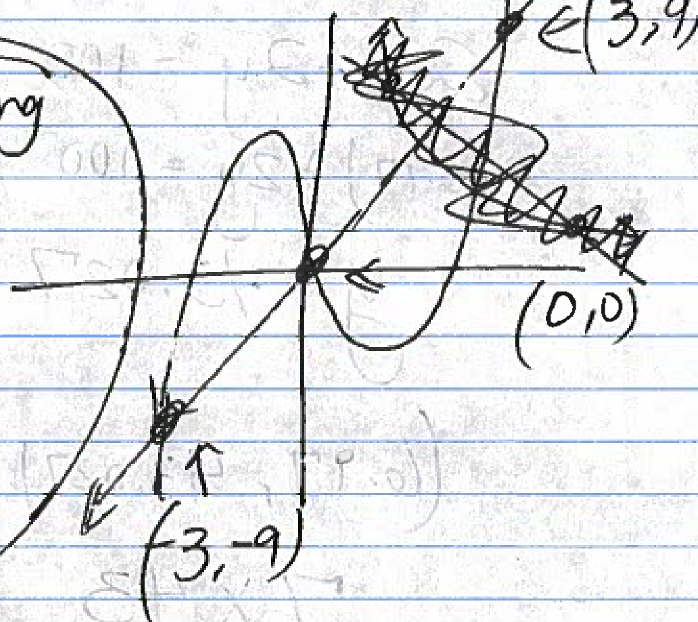
2nd calc 5: intersect

Solve by graphing

$(-3, -9)$

$(0, 0)$

$(3, 9)$



P4 The Elimination Method

1. eliminate a letter to start
2. Solve for remaining letter
3. subst. that value into #1

ie 1

$$3(2x+3y=5) \quad ①$$
$$2(-3x+5y=21) \quad ②$$

① eliminate x & solve for y

$$6x+9y=15$$
$$-6x+10y=42$$
$$19y=57$$
$$\frac{19y}{19}=\frac{57}{19}$$
$$y=3$$

② use y=3 into 1st equation & solve for x

$$2x+3y=5$$
$$2x+3(3)=5$$
$$2x+9=5$$
$$-9 \quad -9$$
$$2x=-4$$
$$x=-2$$

$$(-2,3)$$

5

P5 Finding <u>No solution</u>

ie 1 $2(x-3y=-2)$

$-1(2x-6y=4)$

1 Eliminate x

$$2x-6y=-4$$
$$-2x+6y=-4$$
$$☺ = -8$$

No solution
does <u>NOT</u> equal
same amount

equations end up

$$y=\frac{1}{3}x+\frac{2}{3}$$
$$y=\frac{1}{3}x-\frac{2}{3}$$

same slope

P6 Finding Infinitely Many solutions

on calc.

ic1 $3(4x - 5y = 2)$ rref

$1(-12x + 15y = -6)$

$12x - 15y = 6$

$-12x + 15y = -6$

same values

infinite solutions

Both end up as

$y = \frac{4}{5}x - \frac{2}{5}$

P7 Determining the Equilibrium Price

$p = f(x)$ supply curve

$p = g(x)$ demand curve

Find the equilibrium point for Nike

ie1 $g(x) = 160 - 5x$ ← demand curve

$f(x) = 35 + 20x$

or $p = 160 - 5x$

$p = 35 + 20x$

set equal to each other ✓

$$160 - 5x = 35 + 20x$$
$$-35 \quad +5x \qquad -35 \quad +5x$$

$$\frac{125}{25} = \frac{25x}{25}$$
$$x = 5$$

into demand curve

$160 - 5(5)$

$160 - 25 = \$135$

$(5, 135)$

5 million shoes ↑ at \$135 each sold ↑

7.2 Matrix Algebra

P1 Determining the Order of a Matrix

a) $\begin{array}{c|c} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \end{array}$

Rows X Columns

2×3

find $a_{13} = 3$

row 1, column 3

b) $\begin{array}{c|c} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} & \begin{bmatrix} 1 & -4 \\ 0 & 4 \\ 2 & -1 \\ 3 & 2 \end{bmatrix} \end{array}$

4×2

find $b_{21} = 2$

row 2, column 1

c) $\begin{array}{c|c} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \end{array}$

3×3

P2 Adding & Subracting Matrics

* Must have Exactly same dimensions (#Rows x Columns)

ie 1

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array}\begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 3 & 8 \\ 4 & 2 \\ 6 & 1 \end{bmatrix} \end{array} + \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array}\begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 2 & -5 \\ 1 & 4 \\ 0 & 3 \end{bmatrix} \end{array}$$

3x2 same 3x2

at end

determine matrix size 3x2

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array}\begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 3+2=5 & 8+-5=3 \\ 4+1=5 & 2+4=6 \\ 6+0=6 & 1+3=4 \end{bmatrix} \end{array}$$

ie2 Subtraction

$$\begin{bmatrix} 3 & 6 \\ 4 & 7 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} -1 & -3 \\ -2 & 2 \\ 0 & 5 \end{bmatrix}$$

3x2 same 3x2

3x2

$$\begin{array}{cc} & \begin{array}{cc} C1 & C2 \end{array} \\ \begin{array}{c} R1 \\ R2 \\ R3 \end{array} & \begin{bmatrix} 3-(-1)=4 & 3-(-3)=6 \\ 4-(-2)=6 & 7-2=5 \\ 5-0=5 & 8-5=3 \end{bmatrix} \end{array}$$

3x2 solution

P3 Scalar Multiplication

* mult all numbers by a number outside of matrix

(3)

ie 1

$$3\begin{array}{cc} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} & \\ \begin{bmatrix} 2 & 4 & 1 \\ -1 & 3 & -2 \\ 0 & 7 & 4 \end{bmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array}$$

$$\begin{bmatrix} 6 & 12 & 3 \\ -3 & 9 & -6 \\ 0 & 21 & 12 \end{bmatrix}$$

P4 <u>Matrix Multiplication</u>

* mult. 2 matrices

ie1
$$A = \begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ R_1 & 1, & 2, & 3 \end{array} \quad B = \begin{array}{c|c} & C_1 \\ R_1 & 4 \\ R_2 & 5 \\ R_3 & 6 \end{array}$$

$$\underbrace{1 \times \overbrace{3 \quad 3}^{\text{same}} \times 2}_{\text{solution dimension}}$$

① Find R x C

a) the middle #'s must be the same to mult.

b) the outside # are the new answer dimensions.

② Find solution dimensions 1 x 2

③ Mult Row_1 x $Column_1$

∴ Repeat each Row & column add/subtract b/t each grouping

④ Find the totals

ie1

$$A = \begin{array}{c c} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 2 \end{bmatrix} \end{array} \qquad B = \begin{array}{c c} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{bmatrix} 1 & -4 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \end{array}$$

$2 \times \boxed{3 \quad 3} \times 2$

same

Solution

Step 1 Dimensions

yes

Step 2 solution dimension

2×2

Step 3 Work shown below

$R_1 C_1$

$\cancel{R_1 \times C_1}$ $(2)(1) + 1(0) + (-3)(1)$

$R_1 \times C_2$ ⟶

$R_1 C_2$

$2(-4) + 1(2) + (-3)(0)$

$R_2 C_1$

$0(1) + 1(0) + 2(1)$

$R_2 C_2$

$0(-4) + (1)(2) + 2(0)$

	C_1	C_2
R_1	-1	-9
R_2	2	2

(2x2)

$\boxed{\text{ie2}}$ $A = \begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ R_1 & 2 & 1 & -3 \\ R_2 & 0 & 1 & 2 \end{array}$ $B = \begin{array}{c|cc} & C_1 & C_2 \\ R_1 & 3 & -4 \\ R_2 & 2 & 1 \end{array}$

$2 \times \boxed{3 \quad 2} \times 2$

Step 1: ~~Not the same~~

no solution

~~Identify Inverse Matr~~

ie2 $A = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 2 & 1 & 4 & -1 \end{bmatrix}$ 2×4

$B = \begin{bmatrix} 5 & -1 \\ 0 & 2 \\ -1 & 3 \\ 4 & 2 \end{bmatrix}$

4×2

Answer

2×2

$$\begin{array}{c c} & \begin{array}{cc} \quad C_1 \quad & \quad C_2 \quad \end{array} \\ \begin{array}{c} R_1 \\ \\ R_2 \end{array} & \begin{bmatrix} 1(5)+0(0)+-2(-1)+3(4) & 1(-1)+0(2)+(-2)(3)+3(2) \\ & \\ 2(5)+1(0)+4(-1)+-1(4) & 2(-1)+1(0)+4(3)+-1(2) \end{bmatrix} \end{array}$$

2x2 Answer

$$\begin{bmatrix} 19 & -1 \\ 2 & 8 \end{bmatrix}$$

Day 2

Identity & Inverse Matrices (I_n)

2×2 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

diagonals are 1's from upper left to lower right

3×3 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4×4 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

FORMULA: $AB = BA = I_n$ Long process ?

P6 Verifying an Inverse Matrix

Prove $A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ are inverse matrices

$B = A^{-1}$ $\quad A = B^{-1}$

$$\begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= 6x + 3z = 1$ $\quad 6y + 3w = 0$

$3(2x + 1z = 0)$ $\quad 2y + 1w = 1$

* $6x + 3z = 0$ not same as $6x + 3z = 1$

~~both must~~ equal 1

not inverses

Determinant of a Square Matrix

Formula

2x2 matrix inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

ie1 $A = \begin{array}{cc} a & b \\ \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \\ c & d \end{array}$

$$= \frac{1}{3(2)-(1)(4)}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

every equals zero no inverse

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & -1.5 \end{bmatrix}$$

ie2

$$B = \begin{array}{ccc} a & b & \\ \end{array}$$
$$B = \begin{bmatrix} 1 & 2 & -1 \\ 2^{c} & -1^{d} & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

$\cancel{(1)(-1) - 2(2)}$ $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

2ND matrix edit 3 x 3 fill in

checking to see if denominator equals zero. If not, it has an invers

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

2ND Matrix Math $\boxed{1: det}$ zero matrix $\boxed{A}$ entr entr

$\boxed{-10}$ Not zero so it has an invers

Through hl #41

7.3 Linear Systems: Row Operations

P1 Solving by Substitution

~~(writing one as y=mx+B)~~

ie1

$$x - 2y + z = 7 \quad (1)$$
$$y - 2z = -7 \quad (2)$$
$$\boxed{z = 3} \quad (3)$$

$z = 3$

Substitute into #2 to solve y

$$y - 2(3) = -7$$
$$y - 6 = -7$$
$$+6 \quad +6$$
$$y = -1$$

$y = -1$

$z = 3$

Subst. into #1 to solve for x

$$x - 2(-1) + 3 = 7$$
$$x + 2 + 3 = 7$$
$$x + 5 = 7$$
$$-5 \quad -5$$
$$x = 2$$

$x = 2$

$$\boxed{(\underset{x}{2}, \underset{y}{-1}, \underset{z}{3})}$$

P2 Gaussian Elimination

Step 1: Solve (by eliminating a letter) in 1 & 2

Step 2 solve by eliminating same letter in 2 & 3

Step 3 eliminate a letter from the solutions of 1 & 2 with solutions of 2 & 3

Step 4: Solve for remaining Variables

Step 5 solve for last letter

Step 6 Write answer as

$$(x, y, z)$$

ie1 $x - 2y + z = 7$ ①

$3x - 5y + z = 14$ ②

$2x - 2y - z = 3$ ③

Step 1 $-1(x - 2y + z = 7)$

elimin z $3x - 5y + z = 14$

$-1x + 2y - \cancel{1z} = -7$

$3x - 5y + \cancel{1z} = 14$

$\boxed{2x - 3y = 7}$ A: ① ②

Step 2 $3x - 5y + \cancel{z} = 14$

elimin z $2x - 2y - \cancel{z} = 3$

already opp. value $\boxed{5x - 7y = 17}$ A: ② ③

③

$-5(2x-3y=7)$
$2(5x-7y=17)$

Step 3 eliminate letter X

$-10x+15y=-35$
$10x-14y=34$

$1y=-1$
$y=-1$

Step 4 solve for y using A: 1 : 2 or A: 2 : 3

$y=-1$

$2x-3y=7$
$2x-3(-1)=7$
$2x+3=7$
$-3\quad -3$

$\frac{2x}{2}=\frac{4}{2}$

$x=2$

Step 5 solve for Z Using equation 1

$x - 2y + z = 7$

$x = 2$
$y = -1$

$2 - 2(-1) + z = 7$

$2 + 2 + z = 7$

$4 + z = 7$
$-4 \qquad -4$

$z = 3$

$$\left(\underset{x}{2}, \underset{y}{-1}, \underset{z}{3} \right)$$

ie2

$x - 3y + z = 4$ (1)

$-x + 2y - 5z = 3$ (2)

$5x - 13y + 13z = 8$ (3)

Step 1
1 & 2
eliminate x

$\cancel{x} - 3y + z = 4$

$\cancel{-x} + 2y - 5z = 3$

$\boxed{-1y - 4z = 7}$

Step 2
2 + 3
elimin x

$5(-x + 2y - 5z = 3)$

$5x - 13y + 13z = 8$

$\cancel{-5x} + 10y - 25z = 15$

$\cancel{5x} - 13y + 13z = 8$

$\boxed{-3y - 12z = 23}$

Step 3

A: 1 & 2
with 2 & 3

elimin x

$-3(-1y - 4z = 7)$

$-3y - 12z = 23$

$3y + 12z = -21$

$-3y - 12z = 23$

$0 \neq 2$

No solution
not same

* if you get #'s
that equal ~~equal~~
each other ex. 

infinite
solutions

P3 Solving with Graph Calculator

ie1
$$x - 2y + z = 7$$
$$3x - 5y + z = 14$$
$$2x - 2y - z = 3$$

Augmented matrix (calculator setup)
(use coeffieients)

	C1	C2	C3	C4
R1	1	-2	1	7
R2	3	-5	1	14
R3	2	-2	-1	3

1. 2ND ~~[illegible]~~ matrix edit enter

2. Set up matrix rows : columns

   3x4 for our problem

   type in coeffcients

3 2nd quit

4 2nd Matrix math B↓ rref entr

5 2nd Matrix 1: A 3x4 entr

6 entr

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

last row yours solution

(2, -1, 3)

skip this

ie 2

$$\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array}$$

$$x + 2y - 3z = -1$$

$$R_1$$

$$R_2 \quad 2x + 3y - 4z + w = -1$$

$$R_3 \quad 3x + 5y - 7z + w = -2$$

5 column

coefficient

3 rows

$$\begin{array}{ccccc} 1 & 2 & -3 & 0 & -1 \\ 2 & 3 & -4 & 1 & -1 \\ 3 & 5 & -7 & 1 & -2 \end{array}$$

1. type in 3X5
2. type in coeffients
3. 2ND Quit
4. math rref 2ND matrix A:

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

last row

where is the last letter? W

7.4 Partial Fraction (Factoring Only)

ex $\dfrac{3x-4}{x^2-2x} = \dfrac{3x-2}{x(x-2)}$

P1 Writing the Decomposition Factors

1c1 $\cancel{\dfrac{5x-1}{x^3(x+3)(x^2+1)}}$ $\dfrac{5x-1}{x^2-2x-15}$

$= \cancel{\dfrac{5x-1}{x^3}} = \dfrac{5x-1}{(x-5)(x+3)}$

P2 Decomp a fraction with Repeated Linear Factor

ae7 $$\frac{-x^2+2x+4}{x^3-4x^2+4x}$$

GCF → $$\frac{-x^2+2x+4}{x(x^2-4x+4)}$$

$$\frac{-x^2+2x+4}{x(x-2)(x-2)}$$

P3 Irreducible Quad Factors

$$\frac{x^2+4x+1}{x^3-x^2+x-1}$$

$$= \frac{x^2+4x+1}{(x^3-x^2)+(x-1)} = \frac{x^2+4x+1}{x^2(x-1)(x-1)}$$

#4 Decomp. a Fraction with a Repeated Irreducible Quad Factor

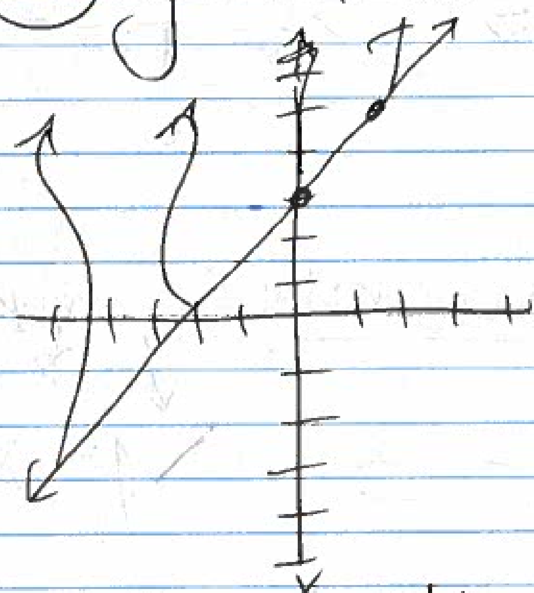
ie1 $$\frac{2x^3 - x^2 + 5x}{(x^2+1)^2} = \frac{x(2x^2 - x + 5)}{(x^2+1)(x^2+1)}$$

$\cancel{(2x+1)(3x+4)}$

7.5 Systems of Inequalities in 2 Variable

P1 Graph a Linear inequality

ie1 $y \geq 2x+3$



$y = mx + B$

m = slope, B = y-int

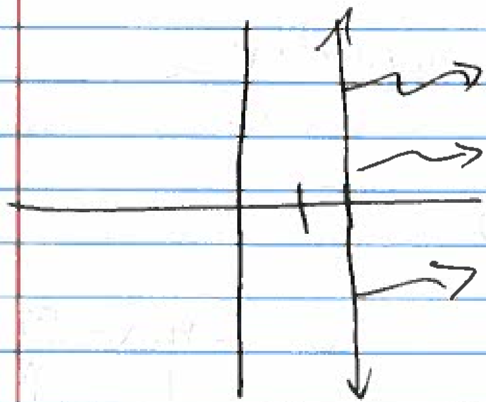
$>$ & $<$ – dotted line

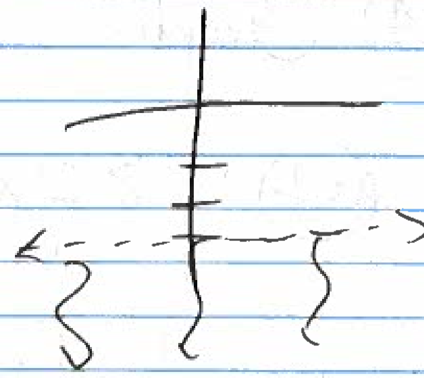
$\geq$ & $\leq$ – solid line

$\left.\begin{matrix} > \\ \geq \end{matrix}\right\}$ up or right

$\left.\begin{matrix} < \\ \leq \end{matrix}\right\}$ down or left ie2 $x \geq 2$

i-1-) $y < -3$



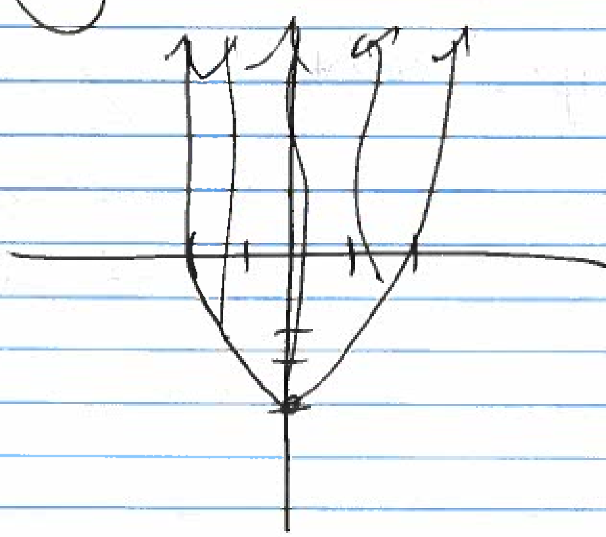


P2 Graph Quadratic

change this to ← mas greater than

ie1 $y \geq x^2 - 3$

$y = x^2 - 3$



Systems of Inequalities

Solution: (x, y) pair that satisfies each inequality

P4 Solving System of Inequalities

ie1 $y > x^2$

$2x + 3y < 4$ Rewrite as $y = mx + B$

$-2x \qquad -2x$

$\frac{3y}{3} < -\frac{2}{3}x + \frac{4}{3}$

$y < -\frac{2}{3}x + \frac{4}{3}$

on calc

ie 2 $2x+y \le 10$

$2x+3y \le 14$

$x \ge 0$

$y \ge 0$

$$\begin{aligned} 2x+y &\le 10 \\ -2x \quad & \quad -2x \\ y &\le -2x+10 \end{aligned}$$

$$\begin{aligned} 2x+3y &\le 14 \\ -2x \quad & \quad -2x \\ \frac{3y}{3} &\le \frac{-2x}{3}+\frac{14}{3} \\ y &\le -\frac{2}{3}x+\frac{14}{3} \end{aligned}$$

on calc